BAYESIAN DECISION-MAKERS REACHING CONSENSUS USING EXPERT INFORMATION

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ABSTRACT

The paper is concerned with the problem of Bayesian decision-makers seeking consensus about the decision that should be taken from a decision space. Each decision-maker has his own utility function and it is assumed that the parameter space has two points, $\Theta = \{\theta_1, \theta_2\}$. The initial probabilities of the decision-makers for Θ can be updated by information provided by an expert. The decision-makers have an opinion about the expert and this opinion is formed by the observation of the expert's performance in the past. It is shown how the decision-makers can decide beforehand, on the basis of this opinion, whether the consultation of an expert will result in consensus.

Keywords: Bayesian decision-makers, expert information

1. INTRODUCTION

In this paper a parameter space with two points, $\Theta = \{\theta_1, \theta_2\}$ and a decision space $D = \{d_1, d_2, \ldots, d_n\}$ are considered. There are decision-makers, $DM_1, DM_2, \ldots DM_k$, who must reach consensus about a decision from decision space where each decision-maker has his own utility function. In their attempt to reach consensus the decision-makers may be able to consult experts and to make use of the information provided by these experts.

As in the papers of DeGroot and Fienberg (1983) and Garisch and Groenewald (1996 and 2007), experts are considered as forecasters where repeated predictions must be made. In this paper the decision-makers have an opinion about each expert and since repeated predictions of the experts are available, this opinion can be formed by looking at the expert's performance in the past. Thus it can be assumed that the decision-makers have a joint opinion about each expert. The view that an expert is not so much a probability assessor, but a person with possibly some special knowledge about the true state of nature, is adopted. This is also the way an expert is defined by Morris (1977). So a “perfect” expert will nominate the true state of nature with probability one.

The view taken in this paper is that the decision-makers must reach consensus about a decision from *D*. Since utility functions are involved, each decision-maker can calculate expected utilities when using the expert and on the basis of these utilities decide beforehand whether to consult the expert or not. It will be shown that the use of certain experts will result in consensus among the decision-makers. By consensus is meant that the same optimal decision is chosen by each decision-maker using his own utility and probability that Θ equals θ_1 as updated by the expert. This decision, whether the consultation of a certain expert will lead to consensus, is based on the opinion held by the decision-makers about the expert. It will be shown that in the process the cost involved in the consultation of the expert and a possible gain by the decision-makers if consensus is reached, are taken into account. Thus the problem is to decide beforehand whether to use an expert or not, because once an expert has given his opinion, it must be used since it is information received.

The information provided by the expert can also be considered as virtual or likelihood evidence that is incorporated in a Bayesian network as discussed in Korb and Nicholson (2004). Other papers on the use of information provided by experts are those of Lindley (1987) and Morris (1974 and 1977). In the paper of DeGroot (1988) the problem of comparing expert opinions is discussed. Papers on the reconciliation and aggregation of probability assessments are those of French (1980 and 1981), Kahn (2004), Lindley (1983) and Winkler (1986). In the paper of Fedrizzi et al. (1995), consensus group decision-making is discussed.

2. UPDATING BAYESIAN BELIEF USING EXPERT OPINION

Consider a single decision-maker, DM_1, and suppose the decision-maker wants to update his prior probabilities of using information provided by an expert. The parameter space of Θ is $\{\theta_1, \theta_2\}$ and suppose that according to, .

Since an expert is considered as a forecaster where repeated predictions must be made, the probability that the expert is correct can be calculated from his performance in the past. Let denote the probability that according to the expert for Suppose DM_1 $P[\Theta = \theta_1] = \pi$

Since an expert is considered as a forecaster where repeated predictions must be made, the probability that the expert is correct can be calculated from his performance in the past. Let $P(V = \theta_i)$ denote the probability that $\Theta = \theta_i$ according to the expert for $i = 1, 2.$ Suppose

$P[V = \theta_1 \mid \Theta = \theta_1] = w_1$, $P[V = \theta_2 \mid \Theta = \theta_1] = 1 - w_1$

$P[V = \theta_1 \mid \Theta = \theta_2] = w_2$ and $P[V = \theta_2 \mid \Theta = \theta_2] = 1 - w_2$.

Now suppose evidence is entered that $V = \theta_1$. Then according to DM_1

$P[\Theta = \theta_1 \mid V = \theta_1] = \alpha P[V = \theta_1 \mid \Theta = \theta_1] P[\Theta = \theta_1] = \alpha\, w_1 \pi$ and

$P[\Theta = \theta_2 \mid V = \theta_1] = \alpha\, w_2 (1 - \pi)$

where $\alpha = P[V = \theta_1] = \dfrac{1}{w_1 \pi + w_2 (1 - \pi)}$.

The posterior belief of can be written as DM_1 can be written as

$$P[\Theta=\theta_1 \mid V=\theta_1]=\frac{a\pi}{a\pi+1-\pi}$$, where $a=w_1/w_2$.

This indicates that it is not the likelihoods w_1 and w_2 that determine the new belief, but rather the ratio $w_1{:}w_2$. Consider $w_1^{'}\neq w_1$ and $w_2^{'}\neq w_2$, then the posterior probability of DM_1 will remain the same if $w_1^{'}/w_2^{'}=w_1/w_2$. It should also be noted that if , $P[\Theta=\theta_1]=1$ the opinion of the expert will have no influence on the posterior probability since $P[\Theta=\theta_1 \mid V=\theta_1]=1$.

Suppose $w_1/w_2=k$ where $w_1+w_2=m\neq 1$. Then $w_1^{'}+w_2^{'}=1$ where $w_i^{'}=w_i/m,\quad i=1,2$ From the previous paragraph it follows that w_1 and w_2 can be replaced by $w_1^{'}$ and $w_2^{'}$ where $w_1^{'}+w_2^{'}=1$ without changing the posterior probability of DM_1 .

Thus without loss of generality it can be stated that

$$P[V=\theta_i \mid \Theta=\theta_i]=w,\quad i=1,2$$ and

$$P[V=\theta_i \mid \Theta=\theta_j]=1-w,\quad j=1,2\quad i\neq j$$

where w and 1 are the probabilities that the expert is correct and incorrect respectively. Thus

$$P[\Theta=\theta_1 \mid V=\theta_1]=\frac{w\pi}{w\pi+(1-w)(1-\pi)}$$

If the prior is uniform, the posterior probability that $\Theta=\theta_1$ is the same as the probability that the expert is correct. Thus if $\pi=0.5$, $P[\Theta=\theta_1 \mid V=\theta_1]=w$. If the probabilities that the expert is correct and incorrect both equal 0.5, no information is obtained from the expert. Thus if $w=0.5$, $P[\Theta=\theta_1 \mid V=\theta_1]=\pi$.

The larger $w/(1-w)$ gets, the higher the opinion about the expert and the more information is expected from him. A small value of $w/(1-w)$ $(w/(1-w)<1)$ means that the expert can still provide a significant amount of information even though he is more likely to be wrong than right.

3. CONSENSUS AMONG DECISION-MAKERS USING AN EXPERT

Consider decision-makers $DM_1, DM_2, \ldots DM_k$,a parameter space, $\{\theta_1,\theta_2\}$ and a decision or action space $D=\{d_1,d_2,\ldots,d_n\}$. Suppose according to DM_i , $P[\Theta=\theta_1]=\pi_i$, $i=1,2,\ldots,k$. Should the decision-makers decide to consult an expert, the posterior probabilities of $DM_1, DM_2, \ldots DM_k$ can be calculated. For DM_i these probabilities are:

$$\pi_i^{'}=P[\Theta=\theta_1 \mid V=\theta_1]=\frac{\frac{w}{1-w}\pi_i}{\frac{w}{1-w}\pi_i+1-\pi_i}$$ and

$$\pi_i^{''}=P[\Theta=\theta_1 \mid V=\theta_2]=\frac{\pi_i}{\pi_i+\frac{w}{1-w}(1-\pi_i)}$$

Suppose each decision-maker has his own utility function. For DM_i this utility function is given in Table 1.

Table 1

		Decisions			
		d_1	d_2	$\ldots$	d_n
Parameter space	θ_1	U_{i11}	U_{i12}	$\ldots$	U_{i1n}
	θ_2	U_{i21}	U_{i22}	$\ldots$	U_{i2n}

Each decision-maker has his own optimal decision. The expected utility of decision d_j according to is DM_i

$$U_i(d_j) = U_{i1j}\pi_i + U_{i2j}(1-\pi_i) \quad i = 1, \ldots, k \quad j = 1, 2, \ldots, n$$

and the decision with maximum utility will be taken. Typically there will be no consensus among the decision-makers about the optimal decision.

In this paper it is suggested that an expert should be consulted by decision-makers if two conditions are satisfied. Firstly each decision-maker must calculate utilities for the use of the expert and according to this decide to use the expert and, secondly, the information provided by the expert must lead to consensus among the k decision-makers about the decision that should be taken from *D*.

Consider the first condition and suppose each decision-maker adds the decision d_{n+1} to his decision space, where d_{n+1} denotes the decision to consult an expert. The utility function of DM_i is now given in Table 2.

Table 2

		Decisions			
		d_1	d_2	$\ldots$	d_{n+1}
Parameter space	θ_1	U_{i11}	U_{i12}	$\ldots$	U_{i1n+1}
	θ_2	U_{i21}	U_{i22}	$\ldots$	U_{i2n+1}

The utilities U_{i1n+1} and U_{i2n+1} must be calculated by to decide whether to consult an expert or not.

There may be a gain if consensus is reached and a cost involved in the consultation of the expert. If e_i denotes the cost, v_i the gain, and $c_i = e_i - v_i$ for , DM_i then U_{i1n+1} and U_{i2n+1} are calculated as follows:

$$U_{i1n+1} = \max_j \{U_{i1j}w + U_{i2j}(1-w)\} - c_i \ , \ j = 1, 2, \ldots, n$$

$$U_{i2n+1} = \max_j \{U_{i1j}(1-w) + U_{i2j}w\} - c_i \ , \ j = 1, 2, \ldots, n$$

If $U_i(d_j)=U_{i1j}\pi_i+U_{i2j}(1-\pi_i)$ and $M_i=\max_j\{U_i(d_j)\}$, $j=1,2,K,n$, then DM_i chooses decision d_{n+1} if $U_i(d_{n+1})\geq M_i$.
Let $D_i=\{o(w)\,|\,U_i(d_{n+1})\geq M_i\}$ and $G=\bigcap_{i=1} D_i$, where $o(w)=\frac{w}{1-w}$, then under the first condition an expert is consulted if $o(w)\in G$.

The second condition for the consultation of an expert is now considered. Before the calculation of U_{i1n+1} and U_{i2n+1} each decision-maker DM_i has his own optimal decision depending on the value π_i where $\pi_i=P[\Theta=\theta_1]$. Suppose d_j is the optimal decision for DM_i if

$$a_{ji}\leq\pi_i\leq b_{ji}\ ,\ \ j=1,2,K,n\ ,\ \ i=1,2,K,k\ .$$

The values of a_{ji} and b_{ji} can be calculated easily for d_j and DM_i .
Let

$G_{ij}=\{l\,|\,U_{i1j}+U_{i2l}\leq U_{i2j}+U_{i1l},l\neq j\}$ and
$H_{ij}=\{l\,|\,U_{i1j}+U_{i2l}>U_{i2j}+U_{i1l},l\neq j\}$.

If $G_{ij}\neq\{\ \}$ and $H_{ij}\neq\{\ \}$,

$a_{ji}=\max\left\{\frac{U_{i2j}-U_{i2l}}{U_{i1l}-U_{i1j}+U_{i2j}-U_{i2l}},l\in H_{ij}\right\}$ and

$$b_{ji}=\min\left\{\frac{U_{i2j}-U_{i2l}}{U_{i1l}-U_{i1j}+U_{i2j}-U_{i2l}},l\in G_{ij}\right\}.$$

If $G_{ij}=\{\ \}$, $b_{ji}=1$ and if $H_{ij}=\{\ \}$, $a_{ji}=0$.

It must now be decided for which experts, in other words for what values of $o(w)$, the posterior probabilities of $DM_1,DM_2,K\,DM_k$ will be updated in such a way that consensus will be reached about the decision that should be taken, irrespective of what the expert says.

If according to the expert $\Theta=\theta_1$, the optimal decision for DM_i is if d_j

$$a_{ji}\leq\frac{\frac{w}{1-w}\pi_i}{\frac{w}{1-w}\pi_i+1-\pi_i}\leq b_{ji}\ ,$$

thus if $\frac{a_{ji}(1-\pi_i)}{\pi_i(1-a_{ji})}\leq o(w)\leq\frac{b_{ji}(1-\pi_i)}{\pi_i(1-b_{ji})}$.

The optimal decision for all the $DM's$ is d_j if

$$\max_{i=1}^{k} a_{ji}'\leq o(w)\leq\min_{i=1}^{k} b_{ji}'\ ,$$

where $a_{ji}^{'} = \dfrac{a_{ji}(1-\pi_i)}{\pi_i(1-a_{ji})}$ and $b_{ji}^{'} \leq \dfrac{b_{ji}(1-\pi_i)}{\pi_i(1-b_{ji})}$.

Thus if $B_j = \left[\max_{i=1}^{k} a_{ji}^{'}, \min_{i=1}^{k} b_{ji}^{'} \right]$,

and according to the expert, $\Theta = \theta_1$, the joint optimal decision for the $DM's$ is d_j if $o(w) \in B_j$.

Similarly it can be shown that if according to the expert $\Theta = \theta_2$, the joint optimal decision for the $DM's$ is d_j if $o(w) \in B_j^{'}$,

where $B_j^{'} = \left[\dfrac{1}{\min_{i=1}^{k} b_{ji}^{'}}, \dfrac{1}{\max_{i=1}^{k} a_{ji}^{'}} \right]$.

Thus, irrespective of what the expert says, consensus will be reached by the decision-makers if $o(w) \in \bigcup A_i$, $\forall i$, where $A_i = B_i \cap \left(\bigcup_{j=1}^{n} B_j^{'} \right)$. To conclude, an expert should be consulted if $o(w) \in \left\{ G \cap \left(\bigcup_i A_i \right) \right\}$.

4. EXAMPLE

The example given in Benjamin and Cornell (1970) will be used as an illustration. In this example, as part of the foundation of a building, a steel section is to be driven down to a firm stratum below ground. The problem is to select a steel pile length when the depth to rock is uncertain. The available actions are driving a 40 ft or 50 ft pile, and the possible states of nature are a 40 ft or 50 ft depth to bedrock. Suppose the contractor and engineer involved in this project must reach consensus about the decision that should be taken, where each has his own utility function. Suppose the utility function of the contractor is given in Table 3 and that of the engineer in Table 4.

Table 3

	d_1	d_2
θ_1	0	-100
θ_2	-400	0

Table 4

	d_1	d_2
θ_1	10	2
θ_2	3	9

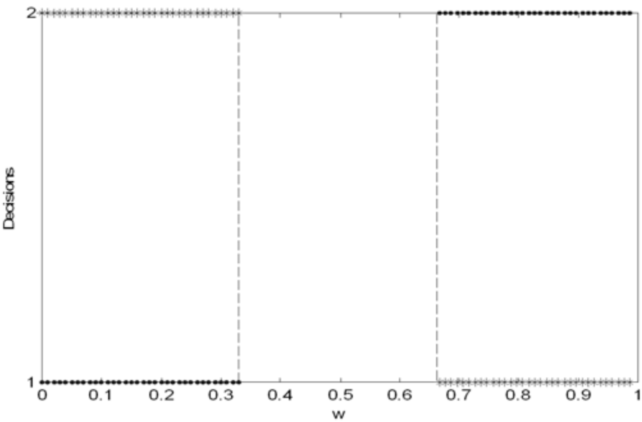
The optimal decision for the contractor is d_1 if the prior probability $\pi_1 \geq 0.8$ and for the engineer if $\pi_2 \geq 0.4286$. Suppose the prior probabilities are $\pi_1 = 0.7$ and $\pi_2 = 0.6$, then decision d_2 is chosen by the contractor, and d_1 by the engineer.

An "expert" is available in the form of an instrument which is used to do a sonic test to give an indication of the depth, and suppose it is known that the instrument is 70% reliable. Thus $w = 0.7$ and $o(w) = 2.3333$. It must be decided by the contractor and the engineer whether this “expert” should be consulted. Suppose for the contractor there is a cost of 10 involved in the consultation of the expert and none for the engineer and the gain in reaching consensus is 5 for the contractor and 1 for the engineer. If d_3 denotes the decision to use this instrument, then it will be the optimal decision for both decision-makers for all possible values of $o(w)$. Thus $2.3333 \in G = [0, \infty]$, and the first condition is satisfied.

Consider now the second condition. Using the matlab package it can be shown that the second condition will be satisfied if $o(w) \in \bigcup A_i = [0, 0.5] \cup [2, \infty]$, thus if $w \in [0, \frac{1}{3}] \cup [\frac{2}{3}, 1]$. Since, the two conditions are satisfied, and the decision-makers should make use of the instrument.

In Figure 1 the consensus decisions are given as a function of From the figure it can be seen that if $w \in [0, \frac{1}{3}]$, the decision-makers will choose d_2 if $\Theta = \theta_1$ according to the expert (denoted by the *'s), d_1 and if the expert indicates otherwise (denoted by the .'s). In the region $w \in [\frac{2}{3}, 1]$, the decision-makers will choose d_1 if $\Theta = \theta_1$ (denoted by the *'s) according to the expert, and d_2 if the expert indicates otherwise (denoted by the .'s). If $w \in (\frac{1}{3}, \frac{2}{3})$ no consensus will be reached.

Figure 1: Consensus decisions



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