LINGUISTICS + MATHEMATICS = TWINS

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ABSTRACT

Language and Mathematics are both so-called "tools" that are used by other disciplines to explain/describe phenomena in those disciplines, but they are scientific disciplines in their own right. Language is a system of symbols, but so is Mathematics. These symbols carry meaning or value. Both originate in the human mind and are then translated into messages of logic. What is important are the relationships between units that are inherent to both disciplines. In practicing the two disciplines, there are elements that correspond. These are *a vocabulary, grammar, a community and meaning.* Psycholinguists and psychologists are interested in the role that language might have in enabling other functions in the human cognitive repertoire. Some argue that language is a prerequisite for a whole range of intellectual activities, including mathematics. They claim that mathematical structures are, in a way, parasitic on the human linguistic faculty. Some evidence for the language: maths connection comes from neurology. Functional imaging studies of the brain show increased activation of the language areas as certain mathematical tasks/challenges are performed. Lesions to a certain part of the brain impair both the linguistic as well as the mathematical ability. We are looking at a fundamentally shared enterprise, a deeply interwoven development of numerical and linguistic aspects. This co-evolution of number concepts and number words suggests that it is no accident that the same species that possesses the language faculty as a unique trait, should also be the one that developed a systematic concept of number.

Key words: Logical Sciences, Symbols, Cognition, Systematic reasoning, Relationships

1. INTRODUCTION

The title of this article may look peculiar to some and even wrong, but they are children of the same parents, Mr. & Mrs. Logic. Both are logical sciences and not that far apart as some people may think. In this article I will try to argue this statement.

Both disciplines could be seen as "tools" used by other disciplines to explain/describe phenomena in those disciplines, but also as scientific disciplines in their own right.

Language is a system of symbols, but so is mathematics too. These symbols carry meaning or value. Both originate in the human mind and is then translated into messages of logic; be it written symbols on paper or used in arguments to explain certain phenomena, thus cognitive sciences. Both disciplines are built on the inherent underlying relations between units.
This brings us to ask:"What is a language?"

To answer the question, we need some definitions of language:

- a systematic means of communicating by the use of sounds or conventional symbols
- a system of words used in a particular discipline
- the code we all use to express ourselves and communicate to others
- a set (finite or infinite) of sentences, each finite in length and constructed out of a finite set of elements

These definitions describe language in terms of the following components:

- A vocabulary of symbols or words
- A grammar consisting of rules of how these symbols may be used
- A community of people who use and understand these symbols
- A range of meanings that can be communicated with these symbols

To expand on the concept of mathematics as a language, we can look at each of these components within mathematics itself.

2. THE VOCABULARY OF MATHEMATICS

Mathematical notation has assimilated symbols from many different alphabets and fonts. It also includes symbols that are specific to mathematics, such as
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Like any other profession, mathematics also has its own brand of technical terminology. In some cases, a word in general usage has a different and specific meaning within mathematics - examples are group, ring, field, category.

In other cases, specialist terms have been created which do not exist outside of mathematics - examples are tensor, fractal, functor. Mathematical statements have their own moderately complex taxonomy, being divided into axioms, conjectures, theorems, lemmas and corollaries. And there are stock phrases in mathematics, used with specific meanings, such as *"if and only if, "necessary and sufficient* and *"without loss of generality".*

3. THE GRAMMAR OF MATHEMATICS

The grammar that determines whether a mathematical argument is or is not valid is mathematical logic. In principle, any series of mathematical statements can be written in a formal language, and a finite state automaton can apply the rules of logic to check that each statement follows from the previous ones.

Various mathematicians (most notably Frege and Russell) attempted to achieve this in practice, in order to place the whole of mathematics on an axiomatic basis. Gedel's incompleteness theorem shows that this ultimate goal is unreachable: any "formal language that is powerful enough to capture mathematics will contain undecidable statements. Nevertheless, the vast majority of statements in mathematics are decidable, and the existence of undecidable statements is not a serious obstacle to practical mathematics.

4. THE LANGUAGE COMMUNITY OF MATHEMATICS

Mathematics is used by mathematicians, who form a global community. It is interesting to note that there are very few cultural dependencies or barriers in modern mathematics. There are international mathematics competitions, such as the International Mathematical Olympiad, and international co-operation between professional mathematicians is commonplace.

5. THE MEANINGS OF MATHEMATICS

Mathematics is used to communicate information about a wide range of different subjects. Here are three broad categories:

- Mathematics describes the real world. Many areas of mathematics originated with attempts to describe and solve real world phenomena - from measuring farms (geometry) to falling apples (calculus) to gambling (probability). Mathematics is widely used in modern physics and engineering, and has been hugely successful in helping us to understand more about the universe around us from its largest scales (physical cosmology) to its smallest (quantum mechanics). Indeed, the very success of mathematics in this respect has been a source of puzzlement for some philosophers.

- Mathematics describes abstract structures. On the other hand, there are areas of pure mathematics which deal with abstract structures, which have no known physical counterparts at all. However, it is difficult to give any categorical examples here, as even the most abstract structures can be co-opted as models in some branch of physics.

- Mathematics describes mathematics. Mathematics can be used reflexively to describe itself this is an area of mathematics called metamathematics. Mathematics can communicate a range of meanings that is as wide as (although different from) that of a natural language.

Over the last few decades evolutionary psychologists and psycholinguists have become increasingly interested in the role that language might have in enabling other functions in the human behavioural and cognitive repertoire. Some have, in fact, argued that language is a prerequisite for a whole range of other intellectual activities, including amongst others, mathematics. According to Varley (2007) some people suggest that the human mind possesses some kind of unique competence that is closely tied to language as we know it. They claim that in cultures where mathematics is applied, the mathematical structures are, in a way, parasitic on this language faculty. Computational procedures and mathematical insights are developed by borrowing tools that are really there to understand and build language. The numbers and the words at our disposal are finite, but what we are able to do with these finite sets is infinite. The master of the English word, Shakespeare, only used 15 000 different words in all his many works, but there are much more than a mere 15 000 sentences.

What makes the alleged link between language and mathematics even more understandable is the fact that we use language to think, to argue and to express ourselves. Our thoughts are executed in language. Some may think in Sesotho, some in Afrikaans and some in English, but we will be able to convey the same message and we will arrive at the same answer in executing mathematics. The well-known and, to some, controversial Sapir-Whorf hypothesis states that "...if you don't have a word for it, you can't think it." Their hypothesis postulates that different language patterns yield different patterns of thought. This idea challenges the possibility of representing the world perfectly with language, because it acknowledges that the mechanisms of any language condition the thoughts of its speaker community. Mathematics often involves the manipulation of symbols, more than numbers; it is what we call abstraction. And so is language too.

Further evidence for the language: mathematics connection comes from neurology. Functional imaging studies of the brain show increased activation of the language areas as certain mathematical tasks/challenges are performed (Varley. 2007). It is a well-known fact that most people with a language disorder called *aphasia,* due to lesions often caused by a stroke, also end up with *acalculia* - a condition that impairs mathematical ability. This may be some proof that there may be some kind of correlation between the capacity to use language effectively and the capacity for doing mathematics effectively. The question that arises is : If a certain section of the brain is damaged, the patient may be unable to draw relationships between symbol and meaning.

Something that puzzled - and interested - philosophers (and linguists and psychologists) through the ages, is the argument that language is the key to human intelligence. Every one of us is constantly confronted with the kaleidoscope of impressions the world presents itself with to us, yet we are able - if we are normal - not only to build a coherent model of the world, but also to recognize relationships of cause and effect, make predictions and develop abstract concepts. Most of us are able to reach this level of cognitive development even before we attend school.

We can thank our verbal nature, along with our fingers, for the ability to develop complex number systems. Some specialists theorise that language and mathematics co-evolved in humans, with language probably emerging just ahead of mathematical concepts. Numbers are not some abstract Platonic entities that must be grasped by humans, but mental tools that we develop ourselves: tools we use to assess properties like cardinality (four buses), rank or ordinal number assignment (the fourth bus), and identity or nominal assignment (the number four bus). Counting words and mathematical concepts, therefore, are intertwined with our language skills, and even appear to be dependent on them. Wiese (2007), a mathematician, builds the case for numerical cognition growing out of the symbolic cognition at the base of language - not as a parasitic spin-off, or a mere naming of numerical concepts, but as an ability whose roots extend to the same underlying cognitive operations. This approach characterizes the human language faculty as a crucial factor in the emergence of systematic numerical thinking. According to this approach, language contributes in three main ways to number development:

(i) Material. Verbal elements, namely the counting sequences of natural languages, are an important instance of numerical tools. It is words used first for iconic cardinality representations, and later as fully blown tools in number assignments that trigger the emergence of number in human development.

(ii) Application. Language as a symbolic system provides a cognitive pattern of dependent linking that can be adapted for the application of words as tools in cardinal, ordinal, and nominal number assignments.

(Hi) Generation. Linguistic recursivity yields words from words and thus allows humans to generate an infinite verbal number sequence.

Taken together, this means that when looking at the emergence of numbers, we are looking at a fundamentally shared enterprise, a deeply interwoven development of numerical and linguistic aspects. This co-evolution of number concepts and number words suggests that it is no accident that the same species that possesses the language faculty as a unique trait, should also be the one that developed a systematic concept of numbers (Heike, 2007).

Most of the mathematical notation in use today was not invented until the 16th century. Before that, mathematics was written out in words, a painstaking process that limited mathematical discovery. Modern notation makes mathematics much easier for the professional, but beginners often find it daunting. It is extremely compressed: a few symbols contain a great deal of information. Like musical notation, modern mathematical notation has a strict syntax and encodes information that would be difficult to write in any other way.

Mathematical language also is sometimes hard for beginners. Words such as *or* and *only* have more precise meanings than in everyday speech. Also confusing to beginners, words such as *open* and *field* have been given specialized mathematical meanings. Mathematical jargon includes technical terms such as *homeomorphism* and *integrable.* But there is a reason for special notation and technical jargon: mathematics requires more precision than everyday speech. Mathematicians refer to this precision of language and logic as "rigor".

Rigor is fundamentally a matter of mathematical proof. Mathematicians want their theorems to follow from axioms by means of *systematic reasoning.* This is to avoid mistaken "theorems", based on fallible intuitions, of which many instances have occurred in the history of the subject. The level of rigor expected in mathematics has varied over time: the Greeks expected detailed arguments, but at the time of Isaac Newton the methods employed were less rigorous. Problems inherent in the definitions used by Newton would lead to a resurgence of careful analysis and formal proof in the 19th century. Today, mathematicians continue to argue among themselves about computer-assisted proofs. Since large computations are hard to verify, such proofs may not be sufficiently rigorous. Axioms in traditional thought were "self-evident truths", but that conception is problematic. At a formal level, an axiom is just a string of symbols, which has an intrinsic meaning only in the context of all derivable formulas of an axiomatic system. It was the goal of Hubert's program to put all of mathematics on a firm axiomatic basis, but according to Cöders incompleteness theorem every (sufficiently powerful) axiomatic system has undecidable formulas; and so a final axiomatization of mathematics is impossible. Nonetheless mathematics is often imagined to be (as far as its formal content) nothing but set theory in some axiomatization, in the sense that every mathematical statement or proof could be cast into formulas within set theory. (Wikipedia)

A strong discipline in linguistics is that of *mathematical linguistics* . It is the study of mathematical structures and methods that are of importance to linguistics. As in other branches of applied mathematics, the influence of the empirical subject matter is somewhat indirect: theorems are often proved more for their inherent mathematical value than for their applicability. Nevertheless, the internal organization of linguistics remains the best guide for understanding the internal subdivisions of mathematical linguistics. The field following the traditional division of linguistics into -+ Phonetics, -+ Phonology, -+ Morphology, -+ Syntax, and -+ Semantics, looking at other branches of linguistics such as - Sociolinguistics or Language Acquisition only to the extent that these have developed their own mathematical methods.

Starting with Bloomfield's (1926, 1933) postulates, the basic conceptual apparatus of mathematical linguistics - in particular, the idea of hierarchical structures composed of relatively stable recurrent items - was developed primarily on the basis of phonological and morphological phenomena. Chomsky (1956, 1959) formulated three theoretical models for the description of linguistic structure, one based on -+ Finite-State Automata (FSA), one based on -+ Context-Free Grammars (CFGs) and one on Context-Sensitive Grammars (CSGs) and/or the even more powerful Unrestricted Rewriting Systems (URSs). The relation between these is investigated under the heading -+ Generative Capacity, and was the basis of much further work on formal language theory within computer science.

Quite interesting is that in normal speech, disciplines of natural sciences also play a role - one that is normally not thought of by speakers and scientists. When a person wants to relay a message, the message originates in his/her brain (psychology), he/she articulates the message by using his/her articulator organs/speech organs (physiology) and produces sound waves (physics). These sound waves travel through the air - not words or sentences. The sound waves then make an impression on the hearing faculty of the listener. From the ear the waves are translated into electrical impulses that travel to the brain to be interpreted as a message.

6. CONCLUSION

Language (or linguistics) and mathematics are different and separate sciences, but are also sister-sciences and are closer linked than what most people would think. Both are - as stated earlier - logical sciences and are therefore also *Human Sciences*.*

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