FIBONACCI NUMBERS AND THE GOLDEN RULE APPLIED IN NEURAL NETWORKS

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ABSTRACT

In the 13th century an Italian mathematician Fibonacci, also known as Leonardo da Pisa, identified a sequence of numbers that seemed to be repeating and be residing in nature (http://en.wikipedia.org/wiki/Fibonacci) (Kalman, D. et al. 2003: 167). Later a golden ratio was encountered in nature, art and music. This ratio can be seen in the distances in simple geometric figures. It is linked to the Fibonacci numbers by dividing a bigger Fibonacci value by the one just smaller of it. This ratio seems to be settling down to a particular value of 1.618 (http://en.wikipedia.org/wiki/Fibonacci) (He, C. et al. 2002:533) (Cooper, C et al 2002:115) (Kalman, D. et al. 2003: 167) (Sendegeya, A. et al. 2007). Artificial Intelligence or neural networks is the science and engineering of using computers to understand human intelligence (Callan R. 2003:2) but humans and most things in nature abide to Fibonacci numbers and the golden ratio. Since Neural Networks uses the same algorithms as the human brain does, the aim is to experimentally proof that using Fibonacci numbers as weights, and the golden rule as a learning rate, that this might improve learning curve performance. If the performance is improved it should prove that the algorithm for neural network's do represent its nature counterpart. Two identical Neural Networks was coded in LabVIEW with the only difference being that one had random weights and the other (the adapted one) Fibonacci weights. The results were that the Fibonacci neural network had a steeper learning curve. This improved performance with the neural algorithm, under these conditions, suggests that this formula is a true representation of its natural counterpart or visa versa that if the formula is the simulation of its natural counterpart, then the weights in nature is Fibonacci values.

Keywords: Neural Networks, Fibonacci Numbers, Golden ratio, Artificial Intelligence

1. INTRODUCTION

1.1 Fibonacci numbers and the Golden Ratio

In the 13th century an Italian mathematician Fibonacci, also known as Leonardo da Pisa, defined Fibonacci numbers releasing it through his model to calculate growth of rabbit populations. Fibonacci numbers being (1, 1, 2, 3, 5, 8, 13…) or

$$F_n = \begin{cases} 0 \rightarrow if\ (n = 0) \\ 1 \rightarrow if\ (n = 1) \\ F_{n-1} + F_{n-2} \rightarrow if\ (n \succ 1) \end{cases} \tag{1}$$

The original problem that Fibonacci investigated in the year 1202 was at what rate rabbits could be breed in ideal circumstances. He noted the following (http://en.wikipedia.org/wiki/Fibonacci) (http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html#spiral)(He, C. et al. 2002:533) (Kalman, D. et al. 2003: 167):

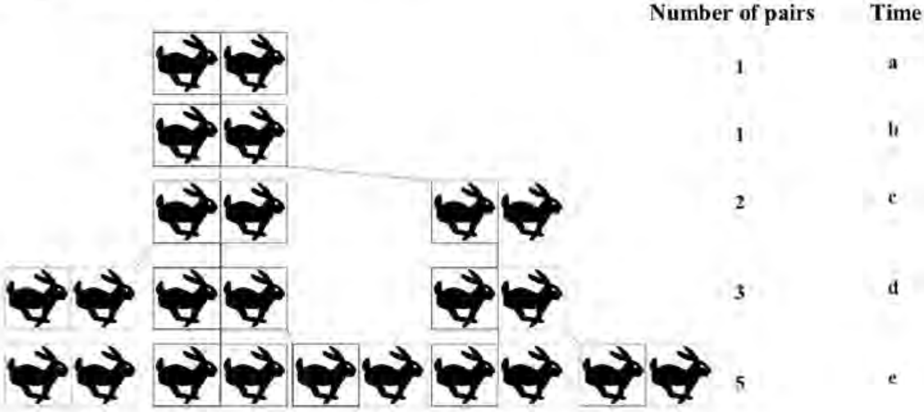

Figure 1: Fibonacci model to for growth of rabbit populations

As seen in figure 1 above, one would start with a single breeding pair, thus the one pair at time 'a'. It takes a while for them to mature enough to be able to reproduce, thus there is still one pair in the time slot 'b'. At the next time instance 'c' the pair produced a new pair ending up in two pairs. The older pair can still reproduce but the new pair is still to young, hence at the next time slot 'd', the older pair produced, were the new pair had none, ending up with three pairs. Then at the time slot 'e', the previously young pair were able to produce, thus there is now the new pair from the previously young pair, a new pair from the original pair, the original pair, the previously young pair and the pair born from the original breeding pair in the previous time slot, resulting in the five pairs at the end of this time slot. Thus he found values recurring were (1, 1, 2, 3, 5, 8, 13 ...) (http://en.wikipedia.org/wiki/Fibonacci) (http://www.webopedia.com/TERM/F/Fibonacci_numbers.html).

An extension of Fibonacci numbers is the golden or Fibonacci ratio. This ratio is encountered in nature, art and music. This ratio is seen when taking the ratios of distances in simple geometric figures. This is linked to the Fibonacci numbers by dividing the bigger Fibonacci value by the one just smaller of it.

For instance, if the ratio of two successive numbers in Fibonacci's series is taken, (1, 1, 2, 3, 5, 8, 13...) and each divided by the number before it, we will find the following series of numbers:

1/1 = 1,	(2)
2/1 = 2,	(3)
3/2 = 1·5,	(4)
5/3 = 1·666...,	(5)
8/5 = 1·6,	(6)
13/8 = 1·625,	(7)
21/13 = 1·61538	(8)

The ratio seems to be settling down to a particular value, which we call the golden ratio or the golden number. It has a value of approximately 1,618034. (http://en.wikipedia.org/wiki/Fibonacci) (He, C. et al. 2002:533) (Cooper, C et al 2002:115) (Kalman, D. et al. 2003: 167) (Sendegeya, A. et al. 2007).

In nature one can observe this in the nautilus shell which epitomizes Fibonacci numbers and the golden ratio and has become the unofficial identity of it.

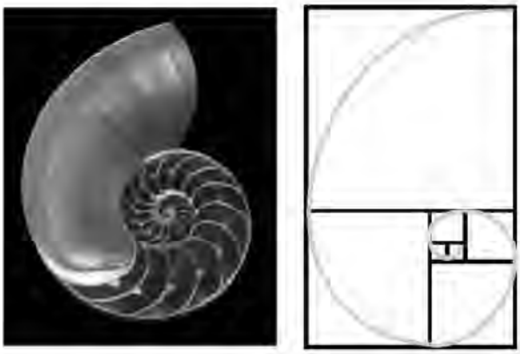

Figure 2: Nautilus shell with its Fibonacci capsulation blocks
(http://www.spirasolaris.ca/cnaut3c.gif)
(http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibInArt.html)

Note how Fibonacci numbers start to repeat in the capsulation block sizes. Fibonacci numbers also resides in flora. Observe the growth points in the following and growth spirals in plants:

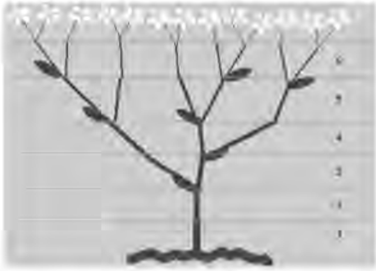

Figure 3: Plant growth point's and plant growth spirals (http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html#spiral)

Similar spirals can be seen in acorns, flowers etc. These values are also present in Honeybees family trees (Basin, S.L. 1963: 55). It can be seen in horns, claws, teeth, shells, spider webs and bacteria multiplying in a Fibonacci sequences (Biggs, B. 2008:165) it is even used in the design of nitrification facilities (Oke, I.A. et al. 2006: 109).

Unconsciously we observe and are exposed to these patterns. This seemed to result in what we preserve as beauty. When observing objects with this it presents a feeling of wellbeing, contentment and gratification. Most Artists would intuitively reproduce this, were other might purposefully use it as tool.

Figure 4: The Mona Lisa 16th-century, oil on a poplar panel. Fibonacci blocks added on to show the relation (http://en.wikipedia.org/wiki/mathematics_and_art).

The Mona Lisa (also known as La Gioconda), painted during the Italian Renaissance by Leonardo Da Vinci. Note the Fibonacci blocks on this famous Mona Lisa. These ratios might accentuate our perception of the focus points. It is also seen in Da Vinci's "The Last Supper".

Fibonacci ratios and numbers are seen in music and instrument ratios. For example a piano has a 5-tone scale, the black notes, and the 8-tone scale, the white notes on the piano. An octave higher is the 13 note that one usually sings or plays in a scale since it sounds more pleasing to the ear. It's also said that Stradivari was aware of the golden section and used it to place the f-holes in his famous violins (Leete, G. 2007:218).

It is found in poetry as seen in this poem called Inspiration Comes of Jim T. Henriksen (http://allpoetry.com/poem/1733415)

1 I
1 am
2 sitting
3 quietly,
5 listening for the
8 quiet noises in the darkness,
13 ghostly images flying between the tall pine trees,
21 illusion, created by the mind, made by shadows, the brain playing tricks on itself.
34 It sits there, the raven, black as night, looking at me with its dark eyes in the dark night. Inspiration comes. Words.

In architecture are Fibonacci ratios not only perceived as beautiful but it is also structurally strong.

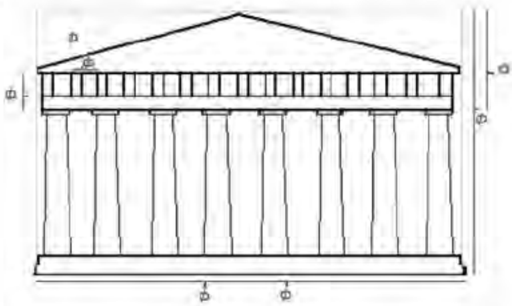

Figure 5: The Parthenon is a temple built for the Greek goddess Athena, the Protectress of Athens, in the 5th century BC on the Athenian Acropolis (http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibInArt.html).

Figure 5 shows the ratios on the Parthenon. These ratios are not only perceived beautiful but it is structurally strong as well, if the age of this structure is considered.
Other buildings include Notre Dame in Paris, Taj Mahal, the United Nations building and the CN Tower in Toronto
(http://goldennumber.net/architecture.htm)(
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibInArt.html).

In humans the ratios can be seen in our geometric build and even in the DNA:

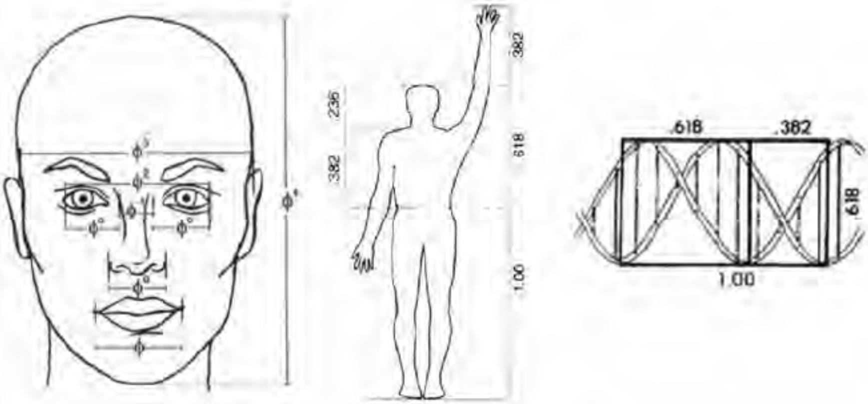

Figure 6: Fibonacci ratios in geometric build and DNA (http://www.stangrist.com/fibonacci.htm).

Figure 6 show that human biology up to the DNA abide to these ratios and multiples of these rations.

1.1 Neural Networks

Artificial Intelligence is based on neural networks. Neural networks are based on the algorithms of a biological brain. Biological brains consist out of neurons which are specialized to carry "messages" through an electrochemical process (Spitzer, M. 2000:1). Neural networks are based on collections of nodes or neurons that are connected in a tree pattern to allow communication between them (Callan R. 2003:2). A single node is a simple processor, computing by combining the input signals with an activation rule to produce an output signal (Callan R. 2003:2).

A single network node would be as follows:

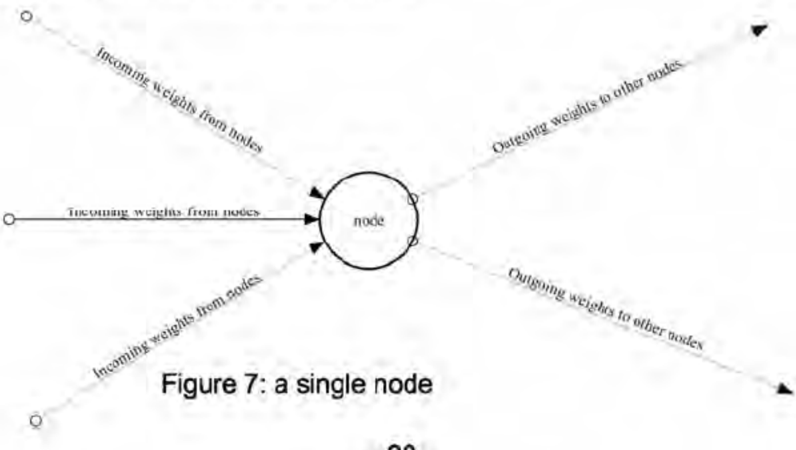

Figure 7: a single node

These nodes are interconnected with weighted connections. A weight is a multiplying constant for the connection's input. Singularly these nodes are limited in operation but by inter-connecting, it gives them the ability to perform complicated tasks. A multilayered network with supervised training is thus capable of learning a required function. This is accomplished by calculating the error at each net or node. The weights are slowly adjusted accordingly to produce all the required outputs.

This process can be mathematically simulated with the formula of the neuron as follows (Callan R. 2003:290):

$$net_j := \sum_{i=1}^{N} x_{i,j} w_{i,j} \tag{9}$$

Where:

- N is the amount of inputs
- i is the node number for a specific input
- j is the number of the net
- x is the input value
- w is weights or constants

This is commonly put through a sigmoid function. The sigmoid is as follows (Callan R. 2003:292):

$$f_j := \frac{1}{1 + \left[e^{\left(-net_j\right)}\right]} \tag{10}$$

Where:

- net is the output of the net
- j is the number of the net

To calculate the error it uses a generalization of the delta rule (Callan R. 2003:294) (Chauvin, Y. et al. 1995:251).
This is accomplished with starting at the last layer with:

$$\delta_j := \left(t_j - o_j\right) o_j \left(1 - o_j\right) \tag{11}$$

Where:

- t is the required output
- o is the net output
- j is the number of the net

The error at the hidden layers is calculated next (Callan R. 2003:303):

$$\delta_j := o_j\left(1 - o_j\right)\sum_k \delta_k w_{j,k} \tag{12}$$

Where
o is the net output
j is the number of the net
k is the number of the net from were the error originate
δ_k is the error from the previous layer
l is the number of that specific path

The weight change for each node is then calculated with (Callan R. 2003:294):

$$\Delta w_{i,j} := \eta \cdot \left(x_{i,j} \cdot \delta_j\right) \tag{13}$$

Where:
η is the learning rate
i is the node number for a specific input
j is the number of the net
x is the input value
δ is the error from the each layer

There after the weights are adjusted as follows (Callan R. 2003:294):

$$W_{i,j} := w_{i,j} + \Delta w_{i,j} \tag{14}$$

Where:
Δw is the weight change
w is the old weights

2. MATERIALS AND METHODS

For this experiment the following neural network was programmed:

Figure 8: 12 input to 8 neurons to 1 neuron with an output

Two identical neural networks as in figure 8 was coded in LabVIEW with the only difference being, that one had random weights and the other (the adapted one) Fibonacci weights. The input data consist of a sine wave and the input nodes were delayed so that the twelve inputs would predict the thirteenth value of the data set.

3. RESULTS

The same data were fed to both networks. Both had the same learning rate (the golden ration) and both had 3000 training cycles.
The result for the Fibonacci weights network after 3000 cycle:

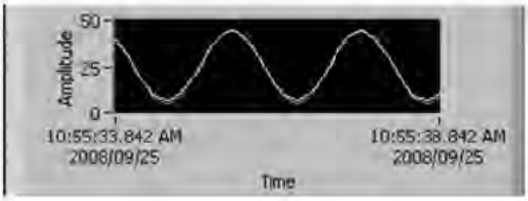

Figure 9: The result for the Fibonacci weights network after 3000 cycle

Figure 9 shows a screen shot of the software for the Fibonacci weights network. In this plot there are two waves the one show the expected results and the other show the output of the network. Note the good fit and a good fit mean that the network successfully studied the expected output.

Figure 10: The result for the random weights network after 3000 cycle

Figure 10 show a screen shot of the random weights software. In this plot there are two waves the one show the expected results and the other show the output of the net work. Note that the two waves don't fit. This means that the network did not successfully study the expected output.

4. CONCLUSION

4.1 Mathematical Conclusion

It was experimentally proven that using Fibonacci numbers as weights and the golden rule as a learning rate one can improve the learning curve performance.

4.2 Philosophical Conclusion

One could assume that since the human biology abides to Fibonacci values and the human brain perceive these numbers and rations as beautiful, that this

Fibonacci values might also be resided in the workings of the human brain.

The improved performance with the above defined algorithm, under these conditions, suggests that the networks formula (as seen in equations 9 to14) is a true representation of its natural counterpart.

The inverse can also be stated, that if the networks formula (as seen in equations 9 to14) is the simulation of its nature counterpart then the weights in nature is Fibonacci values.

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