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A Review of Methods Used to Determine the Overall Stiffness of Unitary Automotive Body Structures

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ABSTRACT

A review of the methods that are used to determine the overall stiffness of automotive body structures was conducted. The review showed that the overall stiffness of body structures is affected by the geometry of the body structure as well as the material used to build the body structure. However, only a limited amount of literature was found that demonstrated how the stiffness behaviour of the body structure is affected by the change in geometry or material. It was also found that the deformation and applied load are directly proportional for body structures that are deformed within the elastic range. However, most of the studies have demonstrated this occurrence by considering the loads that are way less than the approximate, real-life running loads. Therefore, there is a need to study the effects of different materials as well as different geometries on the overall stiffness of body structures when considering the approximate real-life load cases, particularly during the early stages of the development of new vehicle body structures.

Keywords: Automotive body structures, stiffness, bending stiffness, torsion stiffness.

1. INTRODUCTION

From the mid-1930s, the need for developing body structures of passenger vehicles that have reasonable strength and stiffness with minimal weight in shortened development cycles has been well recognized (Swallow, 1939; Booth, 1939; Bastow, 1946; Campbell, 1955; Costin & Phipps, 1965; Cooke, 1965; Fenton, 1976; Brown, et al., 2002; Cavazzuti, et al., 2011). These requirements have not only led to the unification of the chassis frame and the body shell (Swallow, 1939) but, also to the proposals of methods that can be used to model and study the behaviour of body structures under different loading conditions. The objective of these proposed methods is to determine the performance characteristics of body structures such as strength and stiffness with relatively fast turnaround time, particularly during the early stages of the design process.

The body structure of a motor vehicle includes the chassis frame and the body shell (Morello, et al., 2011). The chassis frame is the main assembly on which all the mechanical components such as the powertrain, suspension, transmission and steering systems as well as the body shell, characterized by the relevant and concentrated forces – are mounted (Garrett, et al., 2001; Linton, 2002; Morello, et al., 2011). The body shell is the container for carrying the occupants in comfort and safety as well as luggage safely and without damage (Garrett, et al., 2001), and it can be mounted on the chassis frame or be unitized with it, as in modern vehicles (Morello, et al., 2011). The body structure is designed to fulfil several functions including the following (Linton, 2002; Kumar, et al., 2016; Seward, 2014; Barton & Fieldhouse, 2018):

(i) It must safely carry the maximum load due to the weight of the engine, occupants, fuel tank, drivetrain, and all subsystems of the vehicle,

(ii) It should be able to hold all components together while driving,

(iii) It should be able to protect the occupants against external impact, and also minimise the risk of injury to the other road users during impact.

While fulfilling these requirements, the body structure should have sufficient levels of strength and stiffness so that it can withstand various static and dynamic road loads (Brown, et al., 2002) without suffering from fatigue failure or any other forms of material failure (Barton & Fieldhouse, 2018). The strength of a body structure refers to the maximum running road loads that the body structure can carry without failure (Leckie & Bello, 2009) and is applied to deformation within the elastic range. The stiffness of a body structure on the other hand, is the ability of the body structure to resist flexing or twisting when subjected to running road loads (Sampo, et al., 2010; Leckie & Bello, 2009). The running road loads that are encountered in practice include the following (Gawande, et al., 2018; Brown, et al., 2002; Happian-Smith, 2002):

(i) The weight of the occupants, luggage, body, engine, fuel tank, radiator, drivetrain, and all subsystems of the vehicle,

(ii) The vertical symmetric and asymmetric loads due to uneven ground,

(iii) The lateral loading owing to cornering, nudging a kerb, or steering of the vehicle,

(iv) The fore and aft loading due to braking, acceleration or deceleration, towing, jacking or obstacles,

(v) The torque loading that is transmitted from the transmission system and the driveline of the engine.

These running road loads induce static as well as dynamic stresses and deflections due to the dynamic inertia loads as well as the masses of passengers and various components (Morello, et al., 2011). These deflections are proportional to the properties of the body structure such as the overall bending and torsion stiffness. The stiffness of a body structure has an important influence on the performance characteristics of the vehicle, through vehicle dynamics and ride comfort (Wood, et al., 2014; George & Riley, 2002; Law, et al., 1998; Crocombe, et al., 2010), even on the load carrying capacity of vehicles (Na, et al., 2015). Vehicle dynamics includes factors such as acceleration or deceleration, handling as well as noise (Pang, 2019; Wang, et al., 2014; Pang, et al., 2010; Coox, et al., 2012). Other performance parameters are harshness, steering, and vibration (Masini, et al., 2004; Brown, et al., 2002; Danielsson, et al., 2016) and the ability to absorb energy during impact (Masini, et al., 2004). Adequate stiffness of the body structure also help to prevent issues such as squeak, and rattle and allows the suspension of the vehicle to work efficiently and predictably (Kavarana & Rediers, 2001; Pang, 2019).

It is, therefore, important to minimize these deflections since excessive deflections may lead to insufficient stiffness that can result in unacceptable vibrations (Brown, et al., 2002; Yan, et al., 2018) which can cause the strength of the structural joints to weaken (Yan, et al., 2018; Masini, et al., 2004) as well as causing issues such as under or oversteering of the vehicle (Muley, et al., 2018; Sampo, et al., 2010). Practical examples of very large deflections or insufficient stiffness include issues such as when the floor plan deflects under the passenger's feet or when doors fail to open or close satisfactorily when the vehicle is parked on uneven ground (Happian-Smith, 2002; Brown, et al., 2002). Very large deformations may also cause the windscreen and the backlight glass to crack (Chen, et al., 2012). The body structure that has insufficient stiffness may also be sensitive to fatigue phenomena (Hazimi, et al., 2018; Sampo, et al., 2010). Therefore, it is important to ensure that deflections due to running road loads are not so large as to impair the functions of vehicles (Brown, et al., 2002; Chen, et al., 2012), and to ensure that the body structure is rigid enough to avoid such practical examples and to ensure a high quality performance of the overall vehicle (Pang, 2019).

The stiffness k of a body structure is defined as the ability of the body structure to resist deformation Δ when subjected to load P (Leckie & Bello, 2009). Alternatively, this is defined by the slope ($k = P/\Delta$) of the graph of the load versus deformation and only applies to body structures that are loaded within the elastic range (Brown, et al., 2002). There are usually two different kinds of stiffnesses that are used as benchmarks when assessing the structural performance of vehicles, particularly during the initial stages of the design process. These stiffnesses are referred to as overall stiffnesses of a body structure and arise primarily from bending loads and are known as the bending stiffness, and torsion loads and are known as the torsion stiffness. These two types of stiffnesses, namely bending and torsion are briefly described hereunder.

Bending stiffness k_b, is defined as the relationship between the symmetrical transverse (bending) deflection Δ of a point near the centre of the wheelbase and the factored static loads P of the body structure. This can also be expressed by relating the maximum deflection of the body structure to a single, symmetrically applied load along the centreline of the wheelbases (Brown, et al., 2002). This is usually determined by relating the deflections of the door apertures of the side frame of the body structure to the applied bending loads (Chen, et al., 2012). The bending stiffness mostly affects the door apertures of the side frame as well as the floor panels (Happian-Smith, 2002).

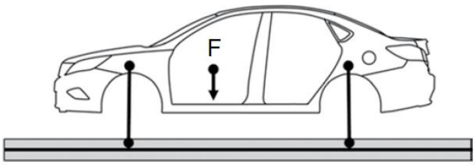
Torsion Stiffness k_t, is defined as the relationship between the torsion induced angle of twist θ of the body structure (measured between the front and rear suspension mounting points) and the pure torsion load T about the longitudinal axis of the body structure (Brown, et al., 2002). Alternatively, the torsion stuffiness of a body structure relates the angle of twist of the body structure to an applied pure torsion load which is applied on the suspension towers at either the front or rear axles, as equal and opposite couples (Chen, et al., 2012). Low values of torsion stiffness mostly affect the doors when the vehicle is parked on uneven ground such as when one wheel is on a kerb (Happian-Smith, 2002) as well as the cornering behavior of a vehicle, when each wheel experiences different vertical loads (Ardigo, et al., 2012). The torsion stiffness is also affected by windscreen and the backlight glass. The torsion stiffness can be reduced by approximately 40% when the glass is removed (Webb, 1984; Happian-Smith, 2002).

The bending and torsion load cases apply completely different local loads to individual component of a vehicle body structure (Brown, et al., 2002). The torsion stiffness has a detrimental effect on the handling characteristics of vehicles (Happian-Smith, 2002) and several studies found that the torsion load case is the most difficult to design for (Mohd Nor, et al., 2016; Brown, et al., 2002; Kirioka, 1965; Cooke, 1965), and therefore the torsion stiffness is often used as a benchmark (Brown, et al., 2002) to indicate the structural efficiency of the vehicle body structure.

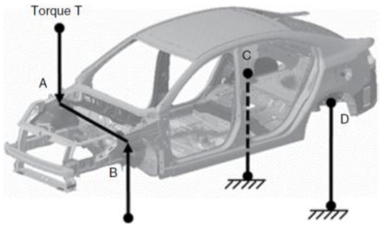
Since the mid-1930s, researchers have often used three techniques to analyse and determine the stiffness of body structures, namely; experimental, analytical and numerical techniques. The goal of this paper is to present an overview of these three methods, with specific interests in how they are used to determine the bending and torsion stiffnesses of unitary automotive body structures. The following section, section 2, discusses these techniques.

2. DETERMINATION OF THE OVERALL STIFFNESS OF BODY STRUCTURES

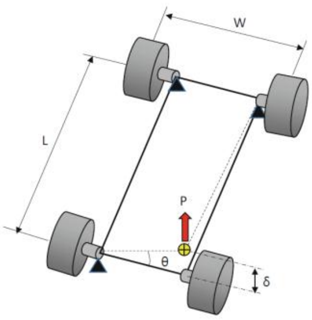
The analysis and determination of the overall stiffness of body structures consist of supporting the body structure at the suspension mounting points, loading it at selected points, and recording the resulting deformations at the point of application of the load. The graph of the applied load versus deformation is then plotted and the stiffness of the body structure is determined as the slope of this graph. Experiments found that the load is directly proportional to the displacement and that the torsion increases linearly with the angle of twist, both for elastic deformation (Swallow, 1939; Scott & Antonsson, 1998; Tebby, et al., 2011; Mohd Nor, et al., 2016; Chen, et al., 2012). Figure 1 shows typical testing diagrams that are used to determine the overall stiffness of body structures. Figure 1(a) depicts a bending load case, whereas, Figure 1(b) and (c) depict pure torsion load cases. Although reasonable values of stiffness are desirable, there is usually a weight penalty that can result in increasing the stiffness of body structures. Therefore, a consideration that is paramount during the preliminary stage of the design process is that of the stiffness to weight ratio parameter (Cooke, 1965; Barton & Fieldhouse, 2018).



(a) Bending stiffness setup



(b) Torsion stiffness setup



(c) Torsion stiffness setup

Figure 1 The testing diagrams of the stiffness of body structures for (a) bending, (b) torsion with equal and opposite loads and (c) torsion applied on one of an axle – courtesy (Pang, 2019; Barton & Fieldhouse, 2018)

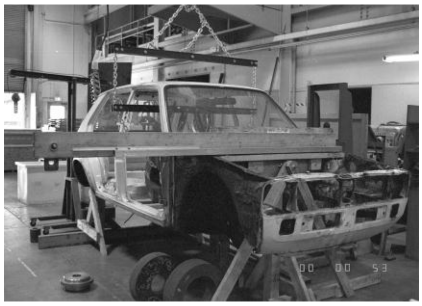
Here, the symbol F stands for the vertical load applied on both the left- and right-hand rocker sills in the case of bending, L the wheelbase, P the applied load, W the track width, θ the angle of twist and δ the vertical displacement both for the torsion load. The symbols W, θ, and δ are shown in Figure 1(c). Figure 1(b) is common in analytical and numerical tests while Figure 1(c) is more common in empirical tests. However, both configurations should yield similar results for the same magnitude of torque imposed in a body structure.

The following subsection, subsection 2.1. discusses the analysis and determination of the bending and torsion stiffness using experimental techniques. The setup for measuring bending deflections is similar to the setup shown in Figure 1(a) and the setup for measuring angles of twist is similar to the setup shown in Figure 1(a) and (b).

3. EXPERIMENTAL TECHNIQUES

In experimental techniques, vehicle body structures are usually scaled down and be modelled using materials such as balsa wood or plastic (Adams, 1993; Mohd Nor, et al., 2016; Thompson, et al., 1998), particularly during the early stages of the design process. However, the actual vehicle body structure is used during the testing phase of the design process (Swallow, 1939; Kirioka, 1965; Podkowski, et al., 2019; Barton & Fieldhouse, 2018). During the testing process, the body structure is subjected to either bending or torsion load in order to examine its behavior. However, these tests can only be carried out on an existing body structure or using models during the rapid prototyping stage of the design process (Podkowski, et al., 2019; Swallow, 1939; Thompson, et al., 1998). Figure 2 shows the empirical setup for stiffness testing of an integral body structure as well as the triangulated space frame.

(a) Bending setup



(b) Torsion setup

(c) Triangulated space frame

Figure 2 Empirical setup for stiffness testing of an integral body structure (a) bending and (b) torsion and (c) torsion of a triangulated space frame – courtesy (Magalhães & Agostinho, 2004; Scott & Antonsson, 1998; Barton & Fieldhouse, 2018)

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The formula used to determine the bending stiffness using the experimental techniques is discussed hereunder.

Bending stiffness – for bending behaviour, testing consists of fixing the body structure to rigid beams running along the length and width of the body structure, the latter at the front and rear axles and subjecting it to a concentrated load at a location close to the rear mounting points of the front seats. Using dynamometers or displacement sensors, the bending resistance is measured in Newtons for each millimeter of transverse displacement that is obtained (Pang, 2019; Swallow, 1939). For the passenger vehicle, the bending stiffness is determined for deflections up till the acceptable limits of deflection of the side frame door apertures (Happian-Smith, 2002).

The bending behaviour of a body structure loaded and restrained as described above is given by Equation 1 thus:

$$k_b = \frac{F}{\delta_{max}} \quad (1)$$

$$where \quad \delta_{max} = \frac{\delta_{lmax}+\delta_{rmax}}{2} - \frac{\delta_{fr}+\delta_{fl}+\delta_{rl}+\delta_{rr}}{2} \quad (2)$$

Here the symbol k_b represents the bending stiffness, F the applied load, δ_{max} the maximum deflection, δ_{lmax} and δ_{rmax} the maximum deflections of the left and right sides rocker sills, respectively, δ_{fr} and δ_{fl} the deflections of the right and left sides of the front mounting points, respectively; and δ_{rl} and δ_{rr} the deflections of the left and right sides of the rear mounting points, respectively (Pang, 2019).

Scott and Antonsson (1998) and Mohd Nor *et al.* (2016) separately performed empirical bending stiffness tests on the body structure of the passenger vehicle shown in Figure 2(b), and on a plastic Simple Structural Surfaces (SSS) model of the passenger vehicle, respectively. The bending stiffness of this body structure was found to be 2500 N/mm. The results of load versus deflection for the test that was conducted by Scott and Antonson (1998) are shown in Figure 3, and the results of load versus deflection for the test that was conducted by Mohd Nor *et al.* (2016) are shown in Figure 4.

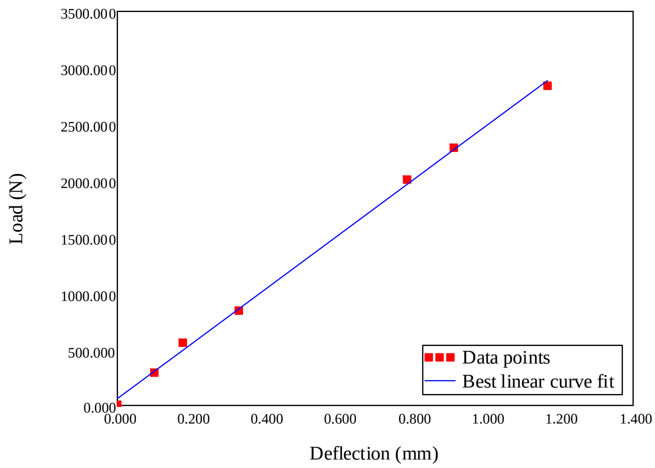

Figure 3 Bending values of load versus deflection results for the body structure shown in Figure 2(b) – courtesy (Scott & Antonsson, 1998)

The panels of the physical SSS model from the study by Mohd Nor *et al.* (2016) were made of plastic polypropylene material. The bending stiffness was determined by Mohd Nor *et al.* (2016) to be 353.8 N/mm. The bending stiffness determined by Scott and Antonsson (1998) is at least 85% higher than that of the study by Mohd Nor *et al.* (2016). This percentage correspond to a ratio of 7.07 between the two stiffness. This ratio is expected given the fact that the Young's modulus of polypropylene is 1 GPa while that of materials that are usually used in the construction of vehicles can range between 69 and 209 GPa. The ratio between the Young's modulus of polypropylene used by Mohd Nor *et al*. (2016) and that of the everyday metals used in the construction of vehicles is at least 69, indicating that Mohd Nor *et al*. managed to obtained a reasonable bending stiffness provided the material used in their study. Although the body structure from and Mohd Nor *et al.* (2016) was constructed from plastic material, the load is still directly proportional to the displacement, at least for smaller displacements that are within the elastic range of the material that are within the elastic range of the material. The challenge with studies carried out by Scott and Antonsson (1998) is that an existing body structure was used, this process is more suitable for evolutionary design where primary load carrying members are constrained by the previous parent vehicle. It is, however, not possible to follow this approach during the design of new vehicle structures since the exact geometry of the structure is not known. Although Mohd Nor *et al.* (2016) was able to demonstrate that the graph of the applied bending load versus deflection of models of body structures made from plastic are linear, the resulting slope or bending stiffness is significantly lower than the target bending stiffness of at least 8000 N/mm for everyday passenger vehicles.

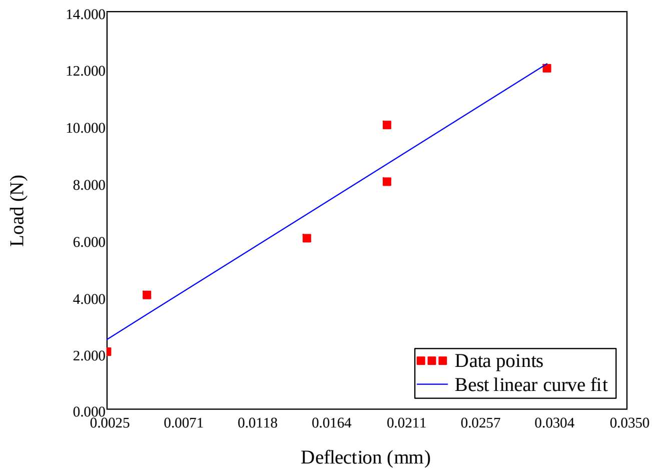

Figure 4 Bending values of load versus deflection for a plastic SSS model that was used by Mohd Nor *et al*. (2016) – courtesy from (Mohd Nor, et al., 2016)

Chen, *et al*. (2012) carried out an experimental study to determine the bending stiffness of a unitary automotive body structure, and the bending load was varied until a maximum load of 10005 N was reached with a resulting vertical deflection of 0.76 mm. The load of 10005 N and the deflection of 0.76 mm resulted in a bending stiffness of 13164.47 N/mm. This bending stiffness is 81% greater than the one obtained by Scott and Antonsson (1998) and it is 97% greater than the one obtained by Mohd Nor *et al.* (2016). Although the bending stiffness obtained by Chen, *et al*. (2012) is very high compared to the one obtained by Scott and Antonsson (1998) and even much higher compared to the obtained by Mohd Nor *et al.* (2016), this bending stiffness is within the target bending stiffness values of every day passenger vehicles and luxury vehicles of 8000 N/mm to 20000 N/mm. Similar to the studies carried out by Scott and Antonsson (1998), the challenge with the studies carried out by Chen, *et al*. (2012) is an existing body structure was used and therefore not suitable of new vehicle designs.

The formula used to determine the torsion stiffness using the experimental techniques is discussed hereunder.

Torsion stiffness - for torsion behaviour, testing consists of fixing the body structure to a rigid transverse beam at the centreline of the rear axle and twisting it by means of a pivoted transverse beam bolted to it at the centre of the front axle. Using spring dynamometers or displacement sensors, the torsional resistance of the body structure is measured in Nm for each degree of twist that is obtained (Swallow, 1939; Pang, 2019).

The torsion behaviour of a body structure that is loaded and restrained as described above is given by Equation 3 thus:

$$k_t = \frac{T}{\theta} \tag{3}$$

$$where \quad T = \left\{\frac{|\mathrm{R_r}|+|\mathrm{R_l}|}{2}\right\} \mathrm{L_s} \tag{4}$$

$$and \quad \theta = \frac{|\delta_\mathrm{r}|+|\delta_\mathrm{l}|}{\mathrm{L_s}} \tag{5}$$

Here the symbol k_t represents the torsion stiffness, T the applied torque, R_r and R_l the right and left side reaction forces, respectively; L_s the lateral distance between the mounting points of the front axle, δ_r and δ_l the deflections of the right and left sides of the front mounting points, respectively, and are determined using dial gauges, sensors or dynamometers, and θ the angle of twist due to these deflections and the lateral distance between the mounting points of the front axle (Tebby, et al., 2011; Thompson, et al., 1998).

A study by Swallow (1939) showed that it is possible to increase the overall torsion stiffness of the body structure by at least 50% while at the same time achieving at least 16% in structural weight savings by substituting the separate chassis-frame by a fully unitary construction, while leaving the vehicles largely identical. Figure 5 shows a comparative study of the torsion stiffness tests of the two body structures represented by a 1938 model of a separate body shell and chassis-frame construction and the 1939 model of a fully unitized construction.

It was noted that the torsion stiffness rose from 934 Nm/deg. (689 lbft/deg.) for the partially unitized construction to 3390 Nm/deg. (2500 lbft/deg.) for the fully unitized construction of body structures. The unusually large hysteresis effect visible in the loading and unloading curve of the partially unitized construction was due to slippage on the mounting points between the body shell and the chassis frame (Brown, et al., 2002). Two things can also be observed from Figure 5. The first is that, the torque is directly proportional to the angle of twist for elastic deformation, that is, the torque-deflection relationship is linear with a zero constant for both the partially and fully unitized body structure, at least for elastic range relationship. Another observation is that, since both vehicles are largely identical, then the subassemblies of components of body structures has a significant impact on the torsional stiffness of the body structure. The study by Swallow (1939) showed that the torsion stiffness of a body structure is mostly affected by its geometry, however, this study also considered existing vehicle body structures.

Another experimental study to determine the torsion stiffness of a unitary automotive body structure was carried out by Chen, *et al*. (2012). They varied the torque from 1020 to 4080 with steps of 1020 until the maximum angle of twist of 0.49 ° was reached at the front of the body structure. The results of the graph of torque versus the angle of twist for the study carried out by Chen, *et al*. (2012) are shown in Figure 6. The overall torsion stiffness of the body structure was then calculated to be 8674.34 N-m/deg. The torsion stiffness obtained by Chen, *et al*. (2012) is 61% greater than the one obtained by Swallow (1939). However, the mass of the unitary body structure used by Chen, *et al*. (2012) was not provided and therefore, it is not possible to compare the stiffness to weight ratio of the body structure used by Chen, *et al*. (2012) and the one used by Swallow (1939). However, the torsion stiffness obtained by Chen, *et al*. (2012) falls within the target stiffness values for typical passenger vehicles of 8000 to 10000 Nm/deg. The study by Chen, *et al*. (2012) demonstrated that the applied torsion is still directly proportional to the angle of twist for actually vehicle body structures and serves as a good basis for validation of other studies, but cannot be used to predict the torsion stiffness of new vehicle structures as it was shown by Swallow (1939) that the geometry of a body structure can significantly affect its torsion stiffness and therefore, the torsion stiffness of body structures can vary significantly depending on their geometries.

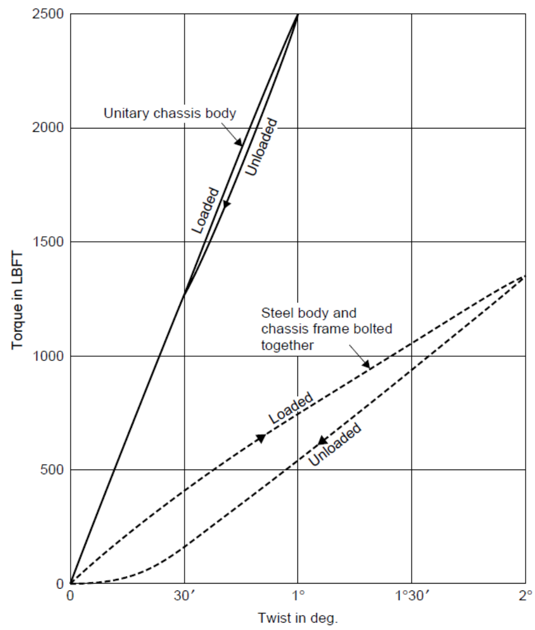

Figure 5 Torsion stiffness tests comparison of steel unitary chassis and steel body and chassis bolted together car body structures of 1938 and 1939 models, respectively – courtesy (Swallow, 1939; Brown, et al., 2002)

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Scott and Antonsson (1998) performed torsion stiffness tests on the body structure of Figure 2(b), with the results shown in Figure 7. The test torsion stiffness of this body structure was found to be 4900 Nm/deg. Mohd Nor *et al.* (2016) conducted torsion stiffness tests on a SSS model of a body structure of passenger vehicles, with the results shown in Figure 8. The panels of the physical SSS model from Mohd Nor *et al.* (2016) were made from plastic polypropylene material. The test torsion stiffness obtained for the work of Mohd Nor *et al.* (2016) was found to be 5.9739 Nm/deg. The torsion stiffness obtained for a real life body structure from the work of Scott and Antonsson (1998) is almost 100% greater than the one obtained on a plastic SSS model by Mohd Nor *et al.* (2016) and it is at least 30% higher than that of Swallow (1939).

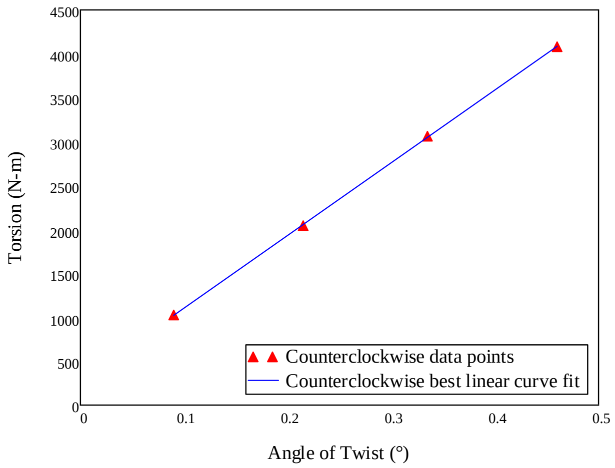

Figure 6 Torsion values of torsion versus angle of twist results for the unitary automotive body structure used by Chen *et al.* (2012) – courtesy (Chen, et al., 2012)

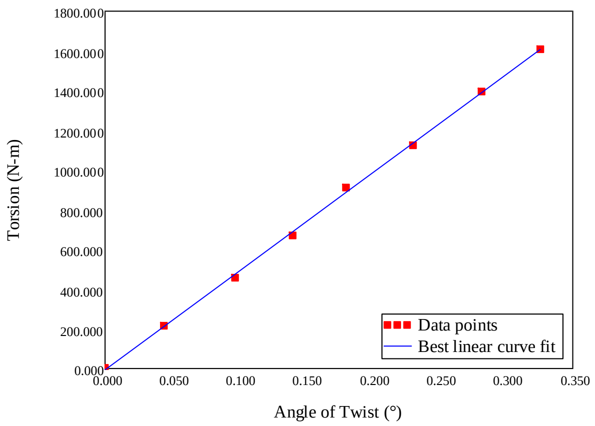

Figure 7 Torsion values of torsion versus angle of twist results for the body structure in Figure 2(b) – courtesy (Scott & Antonsson, 1998)

The body structure that was used by Scott and Antonsson (1998) was that of a 1980 model of a unitary body structure while the one used by Swallow (1939) was that of a 1938 model of a unitary body structure. This is expected given the 40 years difference between the two models, particularly in terms of advances of materials as well as the method of joining different panels of body structures. Though comparison of the stiffness to weight ratio of the two car bodies could not be carried out here as the wheelbase and the track width are not known for both body structures, it is worth noting that the weight of the body structure that was used by Swallow (1939) was about 204 kg and that of Scott and Antonsson (1998) was 170 kg.

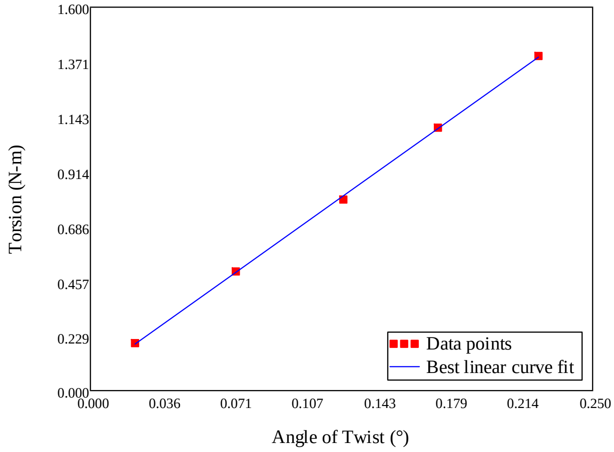

Figure 8 Torsion values torsion versus angle of twist for a plastic SSS model that was used by Mohd Nor *et al.* (2016) – courtesy from (Mohd Nor, et al., 2016)

It is important to note that the more current body structure of Scott and Antonsson (1998) with its stiffness that is 30% greater than the older structure of Swallow (1939) is 17% lighter and is, therefore, a much better body structure in respect of light weighting. The torsion stiffness obtained by Mohd Nor *et al.* (2016) is way lower than both the ones obtained separately by Swallow (1939) and Scott and Antonsson (1998), and even much lower than the one obtained by Chen, *et al.* (2012), and therefore cannot be used as a basis of validation of other studies, particulalry those studies where target values of torsion stiffness of everyday passenger vehicles are required.

While with an exception of the bolted and chassis in the work of Swallow (1939), all the other experimental test results of bending and torsion stiffness discussed so far exhibited directly proportionality between displacement and bending load and angle of twist and torsion load. The main challenge with the studies carried out by Swallow (1939), Scott and Antonsson (1998), and Chen, *et al.* (2012) is that an actually vehicle body structure is required in order to study both the bending and torsion behaviour of the body structure and this has a backlog in a development of new automptive body structures, particularly during the conceptual stages of the design process where an actually body structure is not available. Although the study by Mohd Nor *et al.* (2016) used a simple model to determine both the bending and torsion stiffnesses of a unitary body structure, the obtained bending and torsion stiffnesses are significantly lower than the target bending and torsion stiffness values of everyday unitary body structures, and therefore, there is still a need to develop models or methods that can be used to predict the required target values of these stiffnesses.

The following subsection, subsection 2.2. discusses the analysis and determination of the torsion stiffness using analytical techniques.

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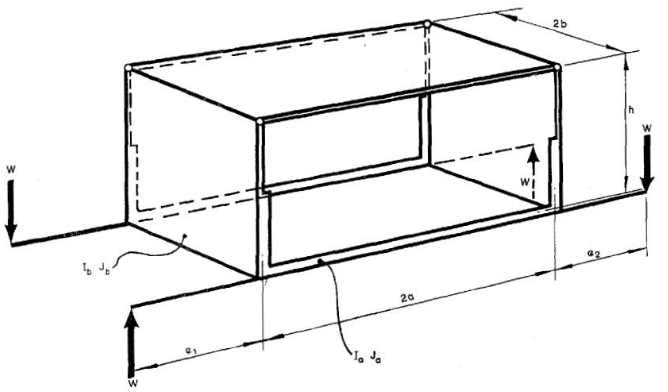
4. ANALYTICAL TECHNIQUES

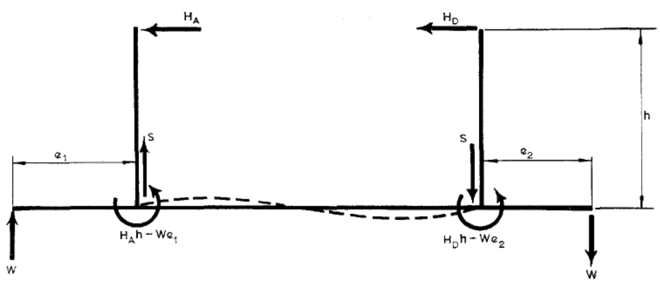
Analytical techniques use procedures such algebra, calculus, differential equations as well as partial differential equations to solve problems through equations (Gerald & Wheatley, 2004). The solutions that arise are referred to as exact solutions. Some of the theories or methods that are used to derive analytical solutions for analysing body structures include the theorem of virtual work or unit load (Argyris & Kelsey, 1960). Other methods include the method of matrix force (Kirioka, 1965), flexure, instability and theory of plastic collapse, St Venant theory of torsion (Fenton, 1976), as well as the strain-energy method (Cooke, 1965).

The method of virtual work as well as the methods of flexure and instability are often used to analyse the side members of body structures. The theory of plastic collapse is often used to analyse the front-end structure of vehicles due to requirements of plastic deformation. The accuracy of the solutions that are obtained from these methods depends on the assumptions that have been made in each case.

These methods include the derivation of complex formulae that must be used at each stage or the use of matrices that must be built largely by hand. This is rather time-consuming and may lead to errors and therefore, more automatic methods must be sought for use when analysing body structures instead (Fenton, 1976). Figure 9 shows Cooke's idealization of a vehicle body structure.



(a) Idealized passenger compartment of a vehicle body structure



(b) Idealized side frame of a vehicle body structure

Figure 9 Cooke's strain energy method representation of a vehicle body structure (a) open box passenger compartment and (b) side frame showing the *A* and *D* posts – courtesy (Cooke, 1965)

The approach used to determine the torsion stiffness using the analytical techniques is discussed hereunder and the formulation is provided in Appendix.

Torsion stiffness – the body structure in torsion is broken down into an open box (the passenger compartment) joined to the input loads at the axle centre lines by rigid beams; Figure 9(a). Using a kinematic diagram, Cooke (1965) determined the distortion of the frame in Figure 9(a) for 1° of overall twist. By solution of the redundant portal frame consisting of *A* post, sill member and *D* post under the two horizontal loads at the roof cantrail, the value of the unknown loads H_A and H_D were determined; Figure 9(b).

A study by Cooke (1965) used the strain-energy theorem to derive the torsion stiffness formula for the passenger compartment section of the body structure. The compartment was approximated as a box section. The torsional stiffness of a passenger compartment of a body structure with the dimensions and section properties illustrated in Figure 9 is given by Equation 11 in Appendix.

Cooke (1965) calculated a torsion stiffness of the passenger compartment of the existing unitary body structure of 31997 Nm/deg. (23600 lbft/deg) against a measured stiffness of 24811 Nm/deg. (18300 lbft/deg.) This difference is large, standing at about 29% and raises questions as to the value of the method, but such discrepancies are expected since the body structure was assumed to have rigid joints in the analytical model which leads to higher values of stiffness such as the one obtained here. The measured torsion stiffness of 24811 Nm/deg. is more than sufficient provided the model year of the vehicle since typical values of the torsion stiffness for modern unitary body structures are approximately 8000 to 10000 Nm/deg for typical passenger vehicles and higher around 12000 to 20000 Nm/deg for luxury vehicles (Brown, et al., 2002; Happian-Smith, 2002; Pang, 2019). However, currently, some vehicles are known to have torsional stiffnesses as high as 28000 Nm/deg. (Pang, 2019).

The advantage of Cooke's (1965) approach is the impression it gives for the expected deformation of the body structure when it is subjected to the torsion load. The disadvantage of the approach is its complicated formulae which must be used at each stage. This is time-consuming and may lead to many arithmetical errors.

The following subsection, subsection 2.3. discusses the analysis and determination of the bending and torsion stiffness using numerical techniques. The setup for measuring bending deflections is similar to the setup shown in Figure 1(a) and the setup for measuring angles of twist is similar to the setup shown in Figure 1(a) and (b).

5. COMPUTATIONAL OR NUMERICAL TECHNIQUES

Numerical models are approximation models. The only procedures that are used by these techniques are arithmetic such as addition, subtraction, multiplication, division and comparison (Gerald & Wheatley, 2004). There is a vast number of numerical techniques that can be used to find solutions by these procedures. Some of the methods that are most commonly

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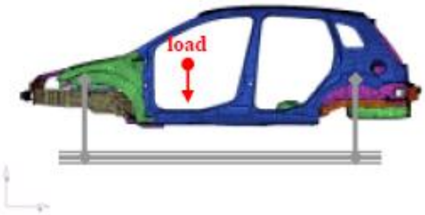
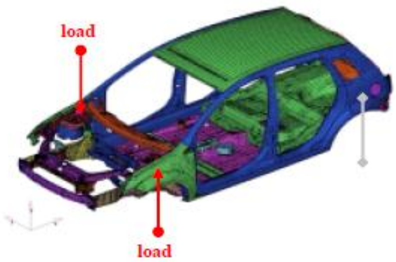
used to find numerical solutions to partial differential equations include finite element method (FEM), finite difference method (FDM), boundary element method (BEM) as well as the finite volume method (FVM) (Venkateshan & Swaminathan, 2014). Each one of these methods is more suitable for certain applications than others. For instance, one advantage the FEM has over FDMs is the relative ease with which boundary conditions of the problem are handled (Burden & Faires, 2011).

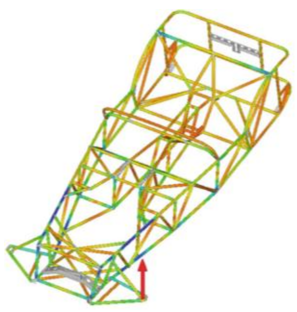
The FEM is now an extremely sophisticated tool for solving numerous engineering and mathematical physics problems and is widely used in practically all branches of engineering for the analysis of structures, solids, and fluids (Fagan, 1992; Bathe, 2014). In the automotive industry, the structural integrity and performance targets of any new vehicle design is thoroughly analysed and evaluated with finite element models, possibly years before the first prototype is built (Fagan, 1992). Some of the commercial Finite Element Analysis (FEA) packages include ABAQUS, ANSYS, Altair, ALGOR, and MSC/Nastran (Rao, 2018; Gerald & Wheatley, 2004). The FEA offers a fast and less expensive approach toward the modelling of body structures (Rao, 2018). Figure 10 shows the FEA setup for stiffness modelling of integral body structures and a triangulated space frame in torsion.



(a) Bending setup



(b) Torsion setup



(c) Triangulated space frame

Figure 10 FEM setup for stiffness modelling of integral body structures (a) bending and (b) torsion and (c) a triangulated space frame in torsion – courtesy (Magalhães & Agostinho, 2004; Barton & Fieldhouse, 2018)

The formulae used to determine the bending and torsion stiffness using the numerical techniques are discussed hereunder.

For bending behaviour, the FEA model is supported at both the front and rear axles and a vertical load is applied on both the left and right rocker sills. The bending resistance is then calculated in Newtons for each millimeter of the maximum displacement that can be measured at the rocker sills (Magalhães & Agostinho, 2004; Kabir, et al., 2017).

The bending behaviour of a body structure that is held rigid by fixed supports at the front and rear axle centrelines and subjected to a concentrated load at a location close to the rear mounting points of the front seats as shown in Figure 10(a) and is given by the equation (Pang, 2019):

$$k_b = \frac{F}{\Delta u} \tag{6}$$

where the symbol k_b represents the bending stiffness, F the applied load, and Δu the maximum deflection of the rocker sills.

For torsion behaviour, numerical analysis requires constraining the FE model with fixed supports at the rear mounting points. The model is then twisted by means of two vertical loads acting in opposite directions at the front axle as shown in in Figure 10(b). The torsional resistance of the model is thereafter calculated in Newton meters for each degree of twist calculated from the vertical displacements on the passenger side and the driver side (Magalhães & Agostinho, 2004; Tebby, et al., 2011; Kabir, et al., 2017).

The torsion behaviour of a vehicle body structure that is held rigid by fixed supports at the rear mounting points and twisted by means of two equal vertical loads acting in opposite directions at the front mounting points is given by the equation (Tebby, et al., 2011):

$$k_t = \frac{T}{\theta} = \frac{FB}{(\theta_d + \theta_p)} \tag{7}$$

$$\text{where } \theta_d = \tan^{-1}\left(\frac{2v_d}{B}\right) \tag{8}$$

$$\text{and } \quad \theta_p = \tan^{-1}\left(\frac{2v_p}{B}\right) \tag{9}$$

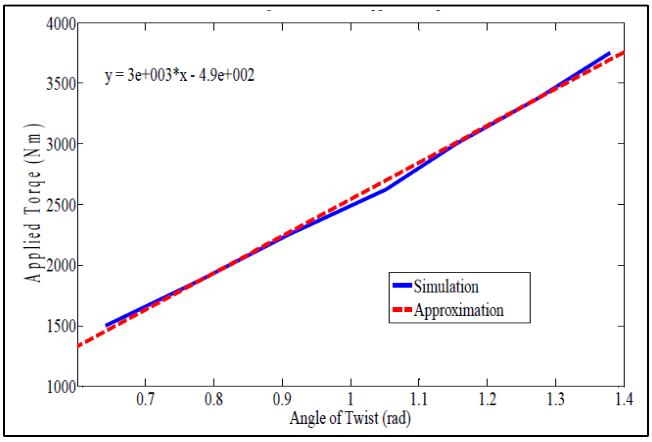
Here the symbol k_t represents the torsion stiffness, T the applied torque, due to the load F that is applied at the mounting points that are separated by the track width of the vehicle, and B and θ the track width and angle of twist, respectively. The angles of twist θ_d and θ_p are based on the track width B and the vertical displacements on the driver side v_d and on the passenger side v_p, respectively.

The torsion behaviour of a body structure that is held rigid by fixed supports at the mounting points and at one side of the front axle and twisted by means of a vertical load on the free side of the front axle is given by the equation (Barton & Fieldhouse, 2018):

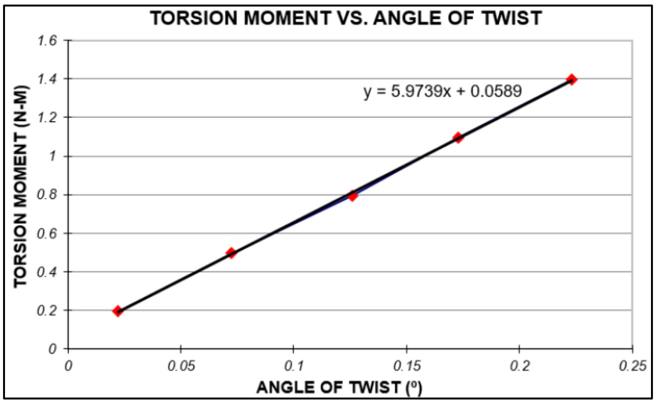
$$k_t = \frac{T}{\theta} = \frac{FB^2}{\delta} \tag{10}$$

where the symbol k_t represents the torsion stiffness, T the applied torque, θ the angle of twist, and δ the vertical displacement measured on the point of application of the torque.

Tebby *et al.* (2011) used two different numerical methods on an integral vehicle body structure that was simplified using the SSS method to determine its torsion stiffness. The structure was made of aluminium and consisted of 18 panels with each panel having a uniform surface thickness of 2 mm. The overall weight of the body structure was approximately 37 kg. They found that the torsion stiffness of the vehicle body structure was 52.36 Nm/deg. (3000 Nm/rad) when using the force method while the displacement method yielded a value of 46.89 Nm/deg. (2686.6 Nm/rad.). These values are too low to be of any practical benefits; they are at least 98% lower than those of Scott and Antonsson (1998). However, both these values are at least 87% higher than those from Mohd Nor *et al*. (2016). The SSS model of Tebby *et al.* (2011) weighed 37 kg which is much lower than the masses of the actual body structures that were used by Swallow (1939) and Scott and Antonsson (1998), and the weight of 37 kg serves as justification for these low torsion stiffnesses. Figure 11 shows a sample comparison of the torsion behavior of a SSS FEM used by Tebby *et al.* (2011) and a physical SSS model used by Mohd Nor *et al*. (2016).



(a) Torsion stiffness of FEM SSS model



(b) Torsion stiffness of a physical SSS model

Figure 11 Comparison between the torsion stiffness of FEM SSS model made out of aluminium (a) and a physical SSS model made out of plastic polypropylene (Tebby, et al., 2011; Mohd Nor, et al., 2016).

A study by Mohd Nor *et al.* (2016) showed that the bending stiffness of a physical SSS model can be reduced by approximately 38% when the windshield panel is removed. The same study showed that the torsion stiffness of a physical SSS model can be reduced by approximately 72% when the windshield panel is removed. Although the reduction in torsion stiffness is substantial, a study by Webb (1984) showed that the torsion stiffness of a typical passenger vehicle can be reduced by approximately 40% if the windscreen is removed. Both the SSS models from Tebby *et al.* (2011) and Mohd Nor *et al.* (2016) were without windscreens. Also, the panels of the physical SSS model from Mohd Nor *et al.* (2016) were made of plastic polypropylene material, which further explains the substantial reduction in the torsion stiffness when the windshield panel is removed.

Despite the fact that the torsion stiffness values of the SSS models used by both Tebby *et al.* (2011) and Mohd Nor *et al*. (2016) are much lower than the target torsion stiffness values of at least 8000 Nm/deg for everyday passenger vehicles, it is observed in Figure 11 that these studies still managed to obtain a linear relationship between the applied torque and the angle of twist. This linear relationship was also demonstrated by the studies carried out on actual vehicle body structures by Swallow (1939), Scott and Antonsson (1998), and Chen, *et al*. (2012). This shows that it is possible to use simple models of vehicle body structures to determine the their stiffnesses. However, it is still not clear if these simple models can be used to approximate the stiffness values of actual vehicle body structures, particularly when the exact geometry of the body structure is not known such as during the conceptual stages of the design process.

6. DISCUSSION AND SUMMARY

The stiffness of a vehicle body structure has an important influence on the performance characteristics such as dynamics and ride comfort as well as the load carrying capacity of the vehicle. The overall bending and torsion stiffnesses are usually used as benchmarks for assessing the structural efficiency of the vehicle body structure. The bending and torsion stiffnesses are often determined using analytical, experimental, or numerical methods. The analytical methods are time-consuming and may to lead to arithmetical errors due to the need to derive complex formulae that must be used at each stage or the need to use matrices that must be built largely by hand.

Experimental methods offer another way of determining the bending and torsion stiffnesses of vehicle body structures by making use of existing body structures. However, making use of existing vehicle body structures is only possible for evolutionary designs and cannot be done during the early stages of developing new concepts of vehicles since the exact geometry of the vehicle is not known. Although it is possible to create physical models of vehicle body structures using materials such as balsa wood or plastic polypropylene, the studies carried out by Mohd Nor *et al.* (2016) on a physical model made out of plastic found the bending and torsion stiffness to be 359.8 N/mm and 5.9739 Nm/deg, respectively.

Table 1. Overall values of stiffness for different passenger vehicle body structures obtained using different methods

	(Swallow, 1939)	(Swallow, 1939)	(Cooke, 1965)	(Scott & Antonsson, 1998)	(Tebby, et al., 2011)	(Chen, et al., 2012)	(Mohd Nor, et al., 2016)
Bending Stiffness (N/mm)	-	-	-	2 500	-	13 164.47	353.8
Torsion Stiffness (Nm/deg.)	934	3 390	31 997	4 900	46.89	8 674.34	5.9739
Mass (kg)	244.9	204.1	-	~170	37	-	-
Material	Steel	Steel	Steel	Different	Aluminium	Different	Polypropylene
Model type	Physical	Physical	Approximate	Physical	SSS FEM	Physical	SSS Physical
Method	Empirical	Empirical	Analytical	Empirical	FEA	Empirical	Empirical

These values of stiffness are very low to be of practical use since the target bending and torsion stiffness of everyday passenger vehicles are at least 8000 N/mm and 8000 Nm/deg, respectively. The challenges with the both the analytical and experimental methods then necessitates the need to seek for and develop alternative methods that can be used to predict the actual target values of stiffness even if only a limited amount of information about the vehicle is limited, such as during the early stages of development of new concepts of vehicles.

One way of achieving this could be by the use numerical methods. Numerical methods are now widely used for solving numerous engineering and mathematical physics problems and are widely used in practically all branches of engineering for the analysis of structures and they give the results that are fairly accurate. These methods offer a fast and less expensive approach toward the modelling of body structures. However, the results obtained from these depends on the assumptions made during the modelling of body structures and may also give the results that are way below the target values of stiffness of vehicle body structures, such as was demonstrated by Tebby *et al.* (2011), and therefore, it is important to consider some of real life situations when making assumptions about the models.

Table 1 shows the stiffness comparison of the studies by Swallow (1939), Scott and Antonsson (1998), Tebby *et al.* (2011), Chen, *et al*. (2012), and Mohd Nor *et al.* (2016).

All these studies showed that the torsion increases linearly with the angle of twist, for vehicle body structures that are loaded within the elastic range. Studies by Tebby *et al.*, (2011) and Mohd Nor *et al.* (2016) showed the effect of material and connectivity of the individual elements of body structure on the overall stiffness of a vehicle body structure to be of significant, since both models were simplified using the SSS method.

The study by Mohd Nor *et al.* (2016) showed that the overall bending stiffness can be reduced approximately by 38% when the windshield panel is removed. Another study by Swallow (1939) showed that it is possible to increase the overall torsional stiffness of the body structure by at least 50% while at the same achieving at least 16% in structural weight savings, by substituting the separate chassis-frame by a fully unitary construction for identical vehicles. This is a typical optimization problem where the objective is to improve stiffness while minimising the weight of the body structure.

From the reviewed literature, it is evident that the stiffness of vehicle body structures can vary significantly depending on their geometries. While with an exception of the physical vehicle structure in the work of Chen, *et al*. (2012) and the approximate analytical model in the work by Cooke (1965), all the other studies determined the of results of bending and torsion stiffness that are far less than the target values of stiffness of at least 8000 N/mm and 8000 Nm/deg for everyday passenger vehicles. Some of these studies determined the values of stiffness without taking into consideration the mass as well as the dimensions of the body structures, making it impossible to compare the determined stiffness versus the mass of the structure. In addition, most of the studies used existing vehicle body structures to study their bending and torsion behavior, which further limit the development of new vehicle concepts with adequate values of stiffness, despite this being one of the major challenges faced by the automotive industry.

Therefore, since there is significant of and a necessity of developing new vehicle body structures that are lightweight and cost effective but have sufficient stiffness both in bending and torsion, therefore, there is a need for further research on the methods that can be used to develop lightweight, stiff, and cost effective vehicle body structures, starting from the packaging requirements, such as during the conceptual stages of the design process.

7. CONCLUSION

The deformation behaviour of body structures was observed to be directly proportional to the applied loads for elastic deformation. The slope of the load versus deformation graph of a vehicle body structure defines the stiffness of the vehicle body structure. It was noted that different running load cases require different definitions of stiffness that are specific to the loads and arising deformation and that the torsion stiffness is most commonly used as a benchmark when developing automotive body structures. It was found that the torsion stiffness is largely affected by the geometry of the body structure and it can be reduced by approximately 40% when a windshield frame is missing. In addition, it was noted that the material that was used to develop body structures has a significant impact on both the weight and the torsion stiffness of body structures.

Although the studies have shown that the load is directly proportional to the deformation of the body structure for vehicle body structures that are deformed within the elastic range, none of the studies has considered the approximate, real life, running load cases when demonstrating this occurrence. Therefore, a study that focuses in determining the overall stiffness of a unitary automotive body structure during the preliminary stages of the design process, taking into account the approximate, real life, running load cases, the mass of the body structure as well as different materials and different geometries still need to be conducted.

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APPENDIX – NOTATION AND FORMULAE FOR COOKE'S ANALYTICAL FORMULA TO DETERMINE TORSION STIFFNESS

Notation

a Half-length of sill.

b Half width of passenger compartment.

e_1 Offset of front wheels forward of front end of passenger compartment.

e_2 Offset of rear wheels aft of rear end of passenger compartment.

H_A Component of H reacted by A post.

H_D Component of H reacted by D post.

h Height of passenger compartment from centre line of sill to centre of cantrail.

I_a Second moment of area of sill cross-section.

I_b Average second moment of area of cross-section of bulkheads.

J_a Polar second moment of area of sill cross-section.

J_b Average polar second moment of area of sill cross-section.

S Shear force in sill associated with the total shear force along transverse edge of roof at unit twist.

W Wheel load reacted by roof and floor in differential shear at unit twist of passenger compartment.

A, B, F, k_A, and $k_D$are the factors that can be obtained from Equations 12 through 19.

e' Average of e_1 and e_2.

E Young's modulus of elasticity.

G Shear modulus of elasticity.

I_d Second moment of area of windscreen pillar cross-section.

I_{wsp} Second moment of area of sill cross-section.

j Length of windscreen pillar.

l is the length of constant section which has second moment of area I_d.

T is the torque to twist through 1°

The formula to determine the torsion stiffness for torque to twist the frame through 1° is given in Equation 11, thus (Cooke, 1965):

$$k_t = \frac{T}{\theta} = \frac{\frac{bh}{6a}(A+B)}{1+\frac{e_1+e_2}{2a}-\frac{h}{a}(k_A+k_D)} + \frac{b^2}{688F\left(\frac{ab^2}{J_aG}+\frac{b^3}{3EI_b}\right)} \quad \text{lb.ft/deg} \tag{11}$$

where

$$\frac{2\pi bh}{360a} = H_A\left(\frac{j^3}{3EI_{wsp}} + \frac{2h^2a}{3EI_a}\right) - H_D\left(\frac{h^2a}{3EI_a}\right) + W\left(\frac{e_2ha}{3EI_a} - \frac{2e_1ha}{3EI_a}\right) \tag{12}$$

$$\frac{2\pi bh}{360a} = H_D\left(\frac{l^3}{3EI_d} + \frac{2h^2a}{3EI_a}\right) - H_A\left(\frac{h^2a}{3EI_a}\right) + W\left(\frac{e_2ha}{3EI_a} - \frac{2e_1ha}{3EI_a}\right) \tag{13}$$

$$H_A = A + k_AW \tag{14}$$

$$H_D = B + k_DW \tag{15}$$

$$W\left[1+\frac{(e_1+e_2)}{2a}\right] = \frac{h}{a}(H_A+H_D) \tag{16}$$

$$T = 2bW \tag{17}$$

$$F = \left(\frac{\frac{a^2b}{J_bG}+\frac{a^3}{3EIa}+\frac{e'ab}{J_bG}}{\frac{b^3}{3EI_b}+\frac{a^3}{3EI_a}+\frac{a^2b}{J_bG}+\frac{ab^2}{J_aG}}\right) \tag{18}$$

$$e' = \frac{e_1+e_2}{2} \tag{19}$$