

A numerical analysis of viscoelastic flow between porous moving walls

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Abstract: A viscoelastic fluid flow between porous moving walls is studied. This study is motivated by flow of biological fluids through veins and arteries with moving walls. Most studies in fluid flow rarely include moving walls. A special analysis of viscoelastic fluid flow between moving walls is considered, the problem was previously considered for Newtonian fluids which resulted in a simple fourth order differential equation. The present study resulted in the fifth-order modified Proudman-Johnson equation. A unique partial differential equation is developed which is then solved numerically by the Simple Iteration Method (SIM). The numerical method is newly developed based on the fixed point theorem. The method agrees with other numerical solutions in the literature. The important results reported are that the effect of varying viscoelastic and the drag-force parameters result in decreasing velocity profiles for both high and low Reynolds numbers. Wall dilation and injection/suction effects affect velocity profiles giving more understanding of the flow of biological fluids in both Newtonian and non-Newtonian fluids.

Key-Words: Viscoelastic, Moving walls, Simple iteration Method, Porous Media.

1 Introduction

The study of fluid flow through channels with moving walls is important due to its practical applications. Applications include movement of biological fluids and hydraulic fluids in machinery. A lot of attention has been focused on fluid flow between parallel walls without movement. A mathematical model based on the work of Majdalani et al. [1] is developed. The flow of a viscoelastic fluid through a channel with moving walls is considered. This result in the modification of the well-known Proudman Johnson equation.

The fluid flow between parallel plates has been widely studied, these studies have advanced the understanding fluid flow under different considerations. These studies include among others the work of Attia [2, 3, 4, 5], who investigated couette fluid flow with injection/suction boundary conditions, power law fluid, uniform suction/suction, dusty fluid flow, pressure gradient, variable properties and Hall currents considerations. The studies between parallel plates have also been studied by Perez et al. [6] who investigated the transfer of heat enhancement by electro-convection. Another study between parallel plate with injection and suction was studied by Hassanzadeh and Mehrabian [7].

The study of non-Newtonian fluids is important as these fluids appear in many practical applications, these include paints, oils, soup, blood, toothpaste etc. Akinshilo [8] studied steady and heat transfer in third grade fluids, Dogonchi and Ganji [9], Anjum et al. [10] investigated the Cattaneo-Christov heat flux effects between parallel plates, this study used the Duan-Rach approach. The flow of Giesekus vis-

coelastic fluid was studied by Mokarizadeh et al. [11]. In this study the momentum equation was solved analytically. Khan et al. [12] investigated flow of Casson fluid flow between parallel plates, they used the Homotopy analysis method. Nejad and Javaherdeh [13] studied power-law between parallel plates, the finite volume method was used to solve the governing equations. In this study the shear thinning and shear thickening was investigated in detail. Sui [14] studied viscoelastic based micropolar fluid flow using the Homotopy analysis method.

The study of fluid flow in porous media has been studied by among others Attia [15, 16, 17] who investigated the effect of porosity on fluid flow between parallel plates with suction/injection, exponential decaying pressure and variable properties. Chaudhary and Sharma [18] investigated three dimensional unsteady fluid flow in porous medium. Ojjela et al. [19] considered variations in permeability and affects fluid flow between parallel plates. The numerical method used is the shooting method with the fourth-order Runge-Kutta methods. Singh and Sharma [20] studied three dimensional fluid flow between porous plates. Other studies in porous media include the work of Tomic et al. [21] and Hassanzadeh and Mehrabian [7].

The study of fluid flow on moving walls has been studied by Majdalani et al. [1] and Makukula et al. [22]. Both of these studies investigated the transverse movement of porous walls between parallel plates. Other studies investigated different movements of plates include the work of Rosca et al. [23], Seth et al. [24] and Satish and Venkatasubbaiah [25].

This paper was motivated by the availability of a

small number of studies that considered a transverse movement of parallel plate in fluid flow. This is applicable in many situations such as biological fluids and in machinery. This study considered a viscoelastic fluid flow between moving walls in porous medium with partial slip boundary conditions.

2 Mathematical formulation

A rectangular domain with laminar, isothermal and incompressible fluid flow is considered. The region is bounded by two permeable transversally moving walls. The governing equations are given as in [1];

$$\begin{aligned} \frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} &= 0, \\ \frac{\partial \hat{u}}{\partial t} + \hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v} \frac{\partial \hat{u}}{\partial \hat{y}} &= -\frac{1}{\rho} \frac{\partial \hat{P}}{\partial \hat{x}} + \nu \nabla^2 \hat{u} - \frac{\nu}{K} \hat{u} \\ &- k_0 \left(\frac{\partial^3 \hat{u}}{\partial t \partial \hat{y}^2} + \hat{u} \frac{\partial^3 \hat{u}}{\partial \hat{x} \partial \hat{y}^2} + \frac{\partial \hat{u}}{\partial \hat{y}} \frac{\partial^2 \hat{u}}{\partial \hat{x} \partial \hat{y}} + \hat{v} \frac{\partial^3 \hat{u}}{\partial \hat{y}^3} \right), \\ \frac{\partial \hat{v}}{\partial t} + \hat{v} \frac{\partial \hat{v}}{\partial \hat{x}} + \hat{u} \frac{\partial \hat{v}}{\partial \hat{y}} &= -\frac{1}{\rho} \frac{\partial \hat{P}}{\partial \hat{y}} + \nu \nabla^2 \hat{v} - \frac{\nu}{K} \hat{v} \\ &- k_0 \left(\frac{\partial^3 \hat{v}}{\partial t \partial \hat{y}^2} + \hat{u} \frac{\partial^3 \hat{v}}{\partial \hat{x} \partial \hat{y}^2} + \frac{\partial \hat{v}}{\partial \hat{y}} \frac{\partial^2 \hat{v}}{\partial \hat{x} \partial \hat{y}} + \hat{v} \frac{\partial^3 \hat{v}}{\partial \hat{y}^3} \right) \end{aligned} \quad (1)$$

where \hat{P}, ρ, ν, t, K and k_0 are the dimensional pressure, density, kinematic viscosity, time, permeability and non-Newtonian parameter. The Boundary conditions are given as

$$\begin{aligned} \hat{u}(\hat{x}, a) &= 0, \quad \hat{v}(a) = -v_w = -\dot{a}/c, \\ \frac{\partial \hat{u}}{\partial \hat{y}}(\hat{x}, 0) &= 0, \quad \hat{v}(0) = 0, \quad \hat{u}(0, \hat{y}) = 0. \end{aligned} \quad (2)$$

The suction/injection coefficient c is the measure of the wall porosity as in [1]. The stream function and the mean flow vorticity is defined as

$$\begin{aligned} u &= \frac{\partial \phi}{\partial y}, \quad v = -\frac{\partial \phi}{\partial x}, \\ \xi &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \end{aligned} \quad (3)$$

By differentiating the x -momentum equation with respect to y , and the y -momentum equation with respect to x and subtracting. Substituting (3) into the result and neglecting insignificant terms we obtain (See also [?])

$$\begin{aligned} \frac{\partial \xi}{\partial t} + u \frac{\partial \xi}{\partial x} + v \frac{\partial \xi}{\partial y} &= \nu \left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} \right) - \frac{\nu}{K} \xi \\ &- k_0 \left(\frac{\partial^3 \xi}{\partial t \partial y^2} + u \frac{\partial^3 \xi}{\partial x \partial y^2} + v \frac{\partial^3 \xi}{\partial y^3} \right). \end{aligned} \quad (4)$$

2.1 The modified Proudman-Johnson equation

A solution of the governing equations can be obtained by considering

$$\begin{aligned} \hat{\phi} &= \nu \hat{x} \hat{F}(y, t)/a, \quad y = \hat{y}/a, \quad \hat{u} = \nu \hat{x} a^{-2} \hat{F}_y, \\ \hat{v} &= -\nu a^{-1} \hat{F}(y, t), \end{aligned} \quad (5)$$

By substituting (5) into (4) we obtain

$$\begin{aligned} \hat{F}_{yyyy} + \alpha \left(y \hat{F}_{yyy} + 3 \hat{F}_{yy} \right) + \hat{F} \hat{F}_{yyy} \\ + \Lambda \left(5 \alpha \hat{F}_{yyyy} - \hat{F}_y \hat{F}_{yyy} + (\hat{F} + \alpha y) \hat{F}_{yyyy} \right) \\ - \hat{F}_y \hat{F}_{yy} - \frac{1}{DaGr} \hat{F}_{yy} \end{aligned} \quad (6)$$

Equation (6) is the modified Proudman-Johnson equation which models wall motion. By following the method considered in [1] we consider the following dimensionless set

$$\phi = \frac{\hat{\phi}}{a\dot{a}}, \quad u = \frac{\hat{u}}{\dot{a}}, \quad v = \frac{\hat{v}}{\dot{a}}, \quad x = \frac{\hat{x}}{a}, \quad F = \frac{\hat{F}}{R}, \quad (7)$$

and

$$\phi = \frac{x F}{c}, \quad u = \frac{x F'}{c}, \quad v = \frac{-F}{c}, \quad c = \frac{\alpha}{R}, \quad (8)$$

equation (6) can be normalized using (7) and (8) to obtain

$$\begin{aligned} F^{IV} + \alpha(y F''' + 3 F'') + R F F''' - (R F' - K_p) F'' \\ + \Lambda (5 \alpha F^{IV} - R F' F''' + (R F + \alpha) F^V) = 0 \\ F''(0) = 0, \quad F(0) = 0, \quad F'(1) = 0, \quad F(1) = 1, \end{aligned} \quad (9)$$

where the primes denote the derivatives with respect to y and $K_p = 1/DaGr$. Equation (9) reverts to the governing equation in [1] when $\Lambda = K_p = 0$

3 Method of solution

In this section we describe the Simple Iteration Method (SIM) based on the fixed point theorem. There are several SIM schemes that are available. Consider an n -nonlinear differential equation of the form

$$\mathcal{F}[y(x), y'(x), \dots, y^n(x)] = 0. \quad (10)$$

Expressing (10) as a sum of its linear and nonlinear terms is given as

$$\begin{aligned} \mathcal{F}[y, y', \dots, y^{(n)}] &= \mathcal{L}[y, y', \dots, y^{(n)}] \\ &+ \mathcal{N}[y, y', \dots, y^{(n)}], \end{aligned} \quad (11)$$

The nonlinear component can be written as the sum of nonlinear terms as

$$\begin{aligned} \mathcal{N} \left[y^{(k)} \right]_{k=0}^n &= y^{(n)} \mathcal{N}_n \left[y^{(k)} \right]_{k=0}^n \\ &+ y^{(n-1)} \mathcal{N}_{n-1} \left[y^{(k)} \right]_{k=0}^{n-1} + \dots + y^{(2)} \mathcal{N}_2 \left[y^{(k)} \right]_{k=0}^2 \\ &+ y^{(1)} \mathcal{N}_1 \left[y^{(1)} \right]_{k=0}^1 + y^{(0)} \mathcal{N}_0 \left[y^{(0)} \right] \end{aligned} \quad (12)$$

The SIM scheme can generally be represented as

$$\begin{aligned} \sum_{k=0}^n a_k(x) y_{r+1}^{(k)} + \sum_{k=0}^n y_{r+1}^{(k)} \mathcal{N}_k \left[y_r^{(i)} \right]_{i=0}^k &= 0, \\ \text{where } \sum_{k=0}^n a_k(x) y_{r+1}^{(k)} &= \mathcal{L} \left[y_{r+1}^{(i)} \right]_{i=0}^n, \end{aligned} \quad (13)$$

$r + 1$ is the current iteration level, y_0 is the initial guess. Applying the Simple Iteration Method (SIM) on (9) we obtain

$$\begin{aligned} \mathcal{L} \left[F^i \right]_{i=0}^5 + F^{(3)} \mathcal{N}_3 \left[F^i \right]_{i=0}^3 \\ + F^{(2)} \mathcal{N}_2 \left[F^i \right]_{i=0}^2 &= 0, \end{aligned} \quad (14)$$

where

$$\begin{aligned} \mathcal{L} \left[F^i \right]_{i=0}^5 &= \Lambda \alpha F^{(5)} + \alpha \left(y F^{(3)} + 3F^{(2)} \right) \\ &+ (1 + 5\Lambda \alpha) F^{(4)} - k_p F^{(2)}, \end{aligned} \quad (15)$$

$$F^{(5)} \mathcal{N}_5 \left[F^i \right]_{i=0}^5 = \Lambda R F, \quad (16)$$

$$F^{(3)} \mathcal{N}_3 \left[F^i \right]_{i=0}^3 = R(F - F^{(1)}), \quad (17)$$

$$F^{(2)} \mathcal{N}_2 \left[F^i \right]_{i=0}^2 = -R F^{(1)}, \quad (18)$$

Applying the Simple Iteration Method on (9) we obtain

$$\begin{aligned} \Lambda(\alpha + R F_r) F^V + (1 + 5\Lambda \alpha) F^{IV} \\ + (\alpha y + R F_r - \Lambda R F_r') F''' \\ + (3\alpha - R F_r' - k_p) F'' &= 0, \end{aligned} \quad (19)$$

$$\begin{aligned} F_{r+1}(0) &= F_{r+1}''(0) = 0, \\ F_{r+1}(1) &= 1, F_{r+1}'(0) = 0. \end{aligned} \quad (20)$$

$$\begin{aligned} a_{1,r} F^V + a_{2,r} F^{IV} + a_{3,r} F''' + a_{4,r} F'' &= 0, \\ F_{r+1}(0) &= F_{r+1}''(0) = 0, \\ F_{r+1}(1) &= 1, F_{r+1}'(0) = 0. \end{aligned} \quad (21)$$

where

$$\begin{aligned} a_{1,r} &= \Lambda(R F_r + \alpha), \quad a_{2,r} = (1 + 5\Lambda \alpha), \\ a_{3,r} &= \alpha y + R F_r - \Lambda R F_r', \\ a_{4,r} &= 3\alpha + R F_r' - k_p, \end{aligned} \quad (22)$$

where $k_p = 1/DaGr$, Applying the spectral collocation and implementing the boundary conditions. By Using the Chebyshev spectral collocation method we consider $N + 1$ collocation points $j = 0, 1, \dots, N$. The interpolating function approximates the unknown functions and its derivatives at the collocation points. The function is given by

$$F_k(y) = \sum_{j=0}^N F_k(y_j) \mathcal{L}_j(y) \quad (23)$$

where

$$\mathcal{L}_j(y_k) = \prod_{j=0, j \neq k}^N \frac{(y - y_k)}{(y_j - y_k)} \quad (24)$$

is the Lagrange interpolating polynomial.

The continuous derivatives are evaluated at the N collocation points y_j for $j = 0, 1, \dots, N$ to obtain the transformation

$$\begin{aligned} \frac{dF_k}{dy} &= \sum_{j=0}^N F_k \frac{d\mathcal{L}_j}{dy}, \\ &= \sum_{j=0}^N F_k \mathbf{D}, \end{aligned} \quad (25)$$

\mathbf{D} is the differentiation matrix and $F_k = [F(y_0), F(y_1), \dots, F(y_M)]$

The solution is obtained by solving the system

$$\mathbf{A} \mathbf{F}_{r+1} = \mathbf{B}, \quad (26)$$

where

$$\mathbf{A} = \mathbf{a}_{1,r} \mathbf{D}^5 + \mathbf{a}_{2,r} \mathbf{D}^4 + \mathbf{a}_{3,r} \mathbf{D}^3 + \mathbf{a}_{4,r} \mathbf{D}^2.$$

4 Results and discussions

In this section we report the results obtained by the Simple Iteration Method (SIM). The method is compared to other results obtained in the literature. The results are in agreement to five decimal places. This shows that the present method is accurate and can be used as an alternative method for solving boundary value problems. The SIM only give the first solution set when $\beta = 1$ in Makukula et al. [22]. The important results that are reported are the effects of the viscoelastic parameter Λ sometimes known as the Deborah number and the inertial drag-force term $K_p = 1/DaGr^{\frac{1}{2}}$ on the velocity profile and the mean flow function $F(y)$. We also report the effects of varying the filtration Reynolds number Re and the wall dilation rate α on the velocity profiles. This analysis is important in understanding the flow of biological fluids in channels that have moving walls. Furthermore in this study, a viscoelastic fluid with low Reynolds numbers is considered which is consistent with most

biological fluids. The values of the Reynolds number considered are $-10 \leq Re \leq 15$. The Viscoelastic parameter considered was $\Lambda = 0, 0.0015, 0.0016$.

The Simple Iteration Method was used to solve the system (9) using number of collocation points $N = 60$. The comparison of the results are shown in Table 1

Table 1: Comparison of the values of $F'(0)$ and $F'''(0)$ of Makukula et al. [22] with the SIM when $Re = -10, \alpha = 4, \Lambda = K_p = 0$.

Makukula et al. [22]		Present (SIM)	
$F'(0)$	$F'''(0)$	$F'(0)$	$F'''(0)$
0.625007396	8.25544612	0.625007397	8.255436809

The effect of varying the viscoelastic parameter Λ , drag-force term K_p , Reynolds number Re and the wall dilation parameter α is illustrated in graphical representations Figures 1 to 5.

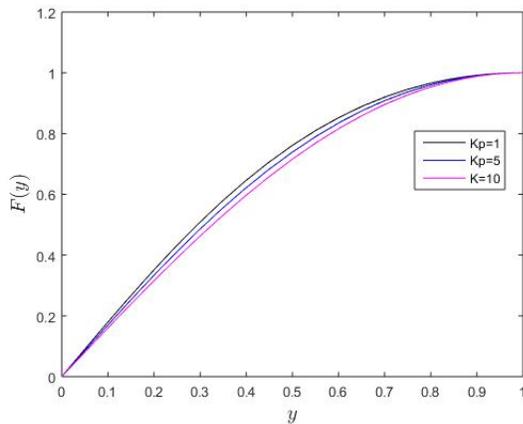


Figure 1: Effects of drag-force parameter K_p on mean-flow function $F(y)$ for $Re = 5, \alpha = 4, \Lambda = 0.0015$

Figure 1 shows the effect of increasing the drag-force parameter on the mean flow function, this results in the reduction if the flow pattern. The results obtained by the Simple Iteration Method (SIM) is consistent with the solution obtained by [1] and [22]. Increasing the drag-force parameter reduce mean-flow function profiles, this is as result of the reduction the flow speed of the fluid.

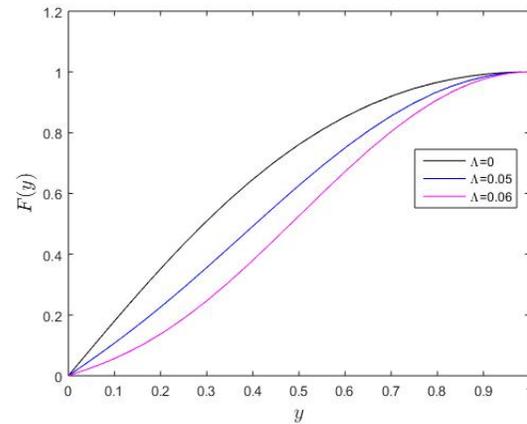


Figure 2: Effects of viscoelastic parameter Λ on mean-flow function $F(y)$ for $Re = 5, \alpha = 4, K_p = 1$

Figure 2 shows the effect of increasing the viscoelastic parameter Λ on the mean flow function, this result in the decrease in the flow pattern. The same pattern was obtained by [22]. Increasing the viscoelastic parameter result in the slowing down of the fluid velocity resulting in the decrease in mean-flow function profiles.

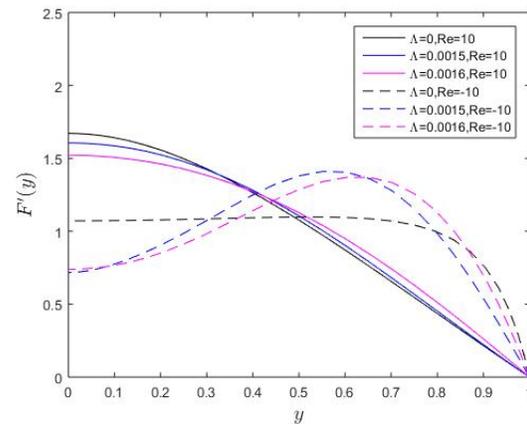


Figure 3: Effects of viscoelastic parameter Λ on velocity profiles for $Re = 10$ (solid lines), $Re = -10$ (dotted lines) $\alpha = 4, K_p = 1$

Figure 3 shows the effect of increasing the viscoelastic parameter Λ on velocity profiles. Increasing the viscoelastic parameter result in the decrease velocity profiles at the center of the channel. The reverse effect noticed towards the surface of the channel is due to the injection of the fluid through the wall $\alpha = 4 > 0$. Lower Reynolds numbers further lower the velocity at the center of the channel. For Newtonian fluids $\Lambda = 0$ a constant velocity profile from the center towards the wall is noted, the decrease of the velocity to zero at the wall is due to the no-slip condition. For low Reynolds numbers an increase in the velocity towards the wall is also caused by the wall dilation.

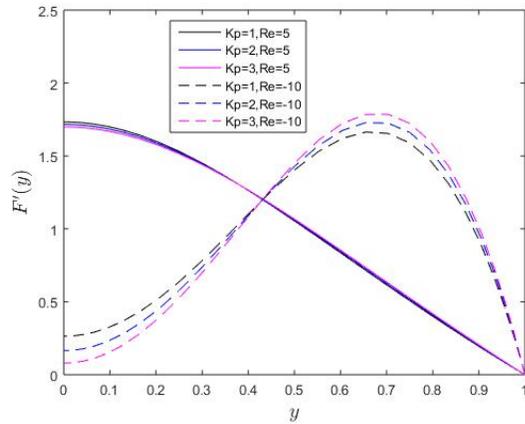


Figure 4: Effects of drag force parameter K_p on velocity profiles for $Re = 10$ (solid lines), $Re = -10$ (dotted lines) $\alpha = 4$, $K_p = 1$

Figure 4 shows the effect of increasing the drag-force parameter K_p on the velocity profiles. Increasing K_p result in the decrease in velocity profiles at the center of the channel. For high Reynolds number $Re = 5$ the variation is small, this is due to the high viscosity of non-Newtonian fluids. The decrease is more enhanced in the case of low Reynolds numbers $Re = -10$. The reverse effect is caused by the fluid injection at the wall.

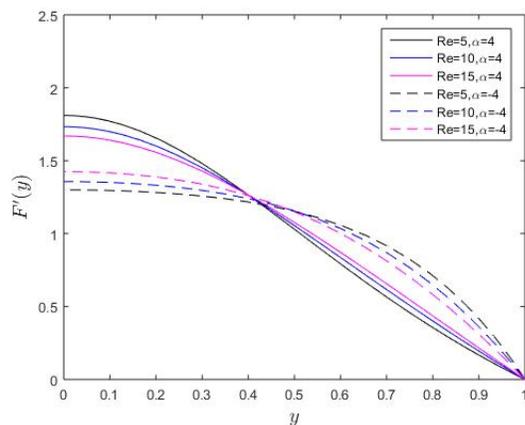


Figure 5: Effects of varying Reynolds number Re on velocity profiles for $\alpha = 4$ (solid lines), $\alpha = -4$ (dotted lines), $\Lambda = 0.0015$

Figure 5 depicts the effect of increasing the Reynolds number on the velocity profiles. Increasing the Reynolds numbers has an effect of increasing the velocity profiles in general. We notice a decrease in the velocity at the center of the channel, this is due to the injection at the walls $\alpha = 4$. This cause the high velocity profiles close to the wall, thereby causing low velocities at the center. For suction $\alpha = -4$ we notice a general decrease in velocity profiles, the fluid in the flow regime is being taken out of the system.

5 Conclusion

In this paper the problem of flow of viscoelastic fluid through moving walls was considered. The movement of biological fluids through moving walls was modelled by a system of partial differential equations. The equations were converted to a fifth order modified Proudman-Johnson equation. The important and interesting results are the effect of varying viscoelastic parameter and the drag-force parameter on velocity profiles. Increasing the viscoelastic parameter result in decreasing the velocity profiles. The variation is small for high Reynolds numbers. For low Reynolds numbers the variation is more enhanced. The results also show a clear distinction of the fluid flow behaviour between Newtonian ($\Lambda = 0$) and non-Newtonian fluids ($\Lambda > 0$) between moving parallel walls. The results also show that increasing the drag-force parameter decrease velocity profiles. The effect of increasing drag-force parameter in the case of low Reynolds numbers decrease the velocity profiles. These results give more understanding of the flow of biological fluids such as blood through veins and arteries. The Simple Iteration Method (SIM) based on the fixed point theorem was shown to be accurate as it was compared to other results in the literature. The paper also reported a new Simple Iteration Method (SIM) scheme which can be adopted by other researchers for solving boundary value problems.

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