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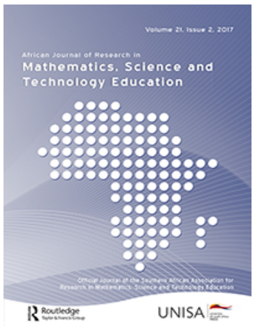

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Original Articles

Probing Factors Influencing Students' Graph Comprehension Regarding Four Operations in Kinematics Graphs

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Students' graph comprehension may be affected by the background of the students who are the readers or interpreters of the graph, their knowledge of the context in which the graph is set, and the inferential processes required by the graph operation. This research study investigated these aspects of graph comprehension for 152 first year undergraduate South African physics students by comparing their questionnaire responses to tasks requiring corresponding mathematics and kinematics graph operations. Interviews with 14 participants aided in interpreting the quantitative data. Participants' gender, year of school completion and study course served as reader characteristics. Their responses were interpreted in terms of their contextual knowledge (understanding kinematics and mathematics concepts, especially graphs, as well as aspects regarding the nature of these subjects) and the inferential processes (visual decoding and judgement) required when performing graph operations. Four graph operations were investigated, namely reading coordinates, determining the gradient of and the area under a line graph and connecting representations. The results of the empirical study indicated that reader characteristics had small to medium effects on the students' responses while their contextual knowledge and the inferential processes largely determined their performance of graph operations. The participants generally transferred their mathematics knowledge on coordinate reading and representations of straight line functions to the kinematics contexts, but not in the cases of parabolic and hyperbolic functions or area under graphs. Insufficient understanding of the gradient concept contributed to weak performances on this graph operation in both the mathematics and kinematics contexts. From this comprehensive study it is deduced that participants' problems with kinematics graphs are mainly due to insufficient contextual knowledge that is foundational to the physical-mathematical model of linear motion represented by the graphs. These deficiencies hamper students in knowing what inferential processes to perform and how to do them. The dependence of the results on the graph contexts and operations reveals the complexity of students' graph comprehension in kinematics.

Keywords: *Graph comprehension; kinematics; mathematics in physics; models*

Introduction

Representational competence is an integral part of physics literacy (Glazer, 2011; Linder, Airey, Mayaba & Webb, 2014). Graphic representations are often used in the analysis of data to determine patterns and relationships between variables and to visually communicate results (Shaughnessy, Garfield & Greer, 1996; Leinhardt, Zalavsky & Stein, 1990). Still, the specific purpose and usage of graphs may differ tremendously, even in subjects as closely related as mathematics and physics.

According to Shah and Freedman (2011), deficiencies in prior content knowledge and graphical skills may retard students' graph comprehension. Students often experience problems with interpreting graphs in the context of physics in general and kinematics in particular. They find it difficult to derive the values of kinematics concepts from the given graphs, to relate one type of kinematics graph to another and to interpret the gradient of a graph or the area under a graph (Beichner, 1994; McDermott, Rosenquist & Van Zee, 1987; Christensen & Thompson, 2012). These difficulties reveal students' problems with associating features of a graph with specific physics concepts. Even students who correctly encode a graph's information may show an inability to map visual features to its physical meaning (Shah & Hoeffner, 2002). Interpretation of kinematics graphs rely heavily on graphical representations that are initially learned in the mathematics context (Leinhardt et al., 1990; Shaughnessy et al., 1996). Unfortunately physics instructors tend to assume that students possess the required mathematics knowledge and skills and expect them to transfer it to physics by themselves, which is often not the case (Meredith & Marrongele, 2008; Glazer, 2011).

Only a few research studies have reported on students' application of mathematics in physics. Most of these studies focussed on problem solving (e.g. Freitas, Jiménez & Mellado, 2004; Tuminaro & Redish, 2004; Redish, 2005; Uhden, Karam, Pietrocola & Pospiech, 2012) or the interpretation of specific aspects such as the gradient of graphs (e.g. Christensen & Thompson, 2012; Woolnough, 2000). No comprehensive study has yet been reported on the effect of reader characteristics, context and inferential processes on four different graph operations set qualitatively and quantitatively in both mathematics and kinematics contexts. With this study we intended to contribute in this regard by investigating factors influencing a group of introductory physics students' performances of graphs in mathematics and kinematics with similar graph operations. In this study, reader characteristics are biographical details of students such as their gender, tertiary study course and year of school completion that may have an effect on their contextual knowledge and inferential processes. Context encompasses both content knowledge (e.g. concepts and their relations) and discernment of aspects regarding the nature of the context the graph is set in (e.g. kinematic graphs with time as independent variable can only be in the first and fourth quadrants of the Cartesian coordinate system). The inferential processes considered for graph comprehension are the visual decoding of the graph and judgement of the graph processes.

Various reasons have been reported in research studies to explain why introductory university students in science-related fields experience difficulty with identifying and applying the fundamental mathematical concepts that they have previously learned. Tuminaro and Redish (2004) have provided evidence that students' inability to transfer their existing mathematics knowledge to physics is the major cause of students' errors. Other researchers found the problem with students' interpretation of graphs in science questions to lie with their deficiencies in mathematical knowledge rather than the transfer of mathematics to the science context (Potgieter, Harding & Engelbrecht, 2008; Scott, 2012). Students may also have conceptual difficulties with the physics concepts as follows from studies of kinematics graphs by researchers such as Beichner (1994) and McDermott et al. (1987).

While the above-mentioned studies were more focussed, the research reported here considered three aspects of graph comprehension, namely reader characteristics, the context and the inferential processes (visual decoding and judgement) required in kinematics graph operations. These aspects of graph comprehension were proposed by Friel, Curcio and Bright (2001). However, the study of Friel et al. (2001) referred to statistical graphs while comprehension of kinematics graphs as physical-mathematical models of motion require different perspectives with regard to the context and inferential processes involved. From this theoretical framework of graph comprehension, the empirical study reported here intended to shed more light on reasons underlying physics students' problems with operating kinematics graphs.

Kinematic Graphs as Physical-mathematical Models

Kinematics graphs are physical-mathematical models that represent motion of objects, normally under the assumption of constant acceleration. Such models result from mathematization of a physical model or system by integration of conceptual physical knowledge with structural and technical knowledge from mathematics (Redish & Kuo, 2015, Uhden et al., 2012). Redish and Kuo (2015) stress that physics students need to gain experience with the physical meaning making with mathematics. This requires them to cue relevant mathematics and physics subsets of knowledge and associate physical meaning to the mathematical symbols and formats. For example, in mathematics the concept of function combines two different symbolic systems, namely algebraic and graphical representations (Leinhardt et al., 1990). The same is true for graphs in physics, but both algebraic and graphical representations are embedded in physical contexts in which the variables have physics meaning.

Lemmer and Gunstone (2016) argued the importance of introducing learners to physical models of real-world phenomena before deriving physical-mathematical models thereof. Sequences containing real-world and laboratory experiences are required to acquaint students with the conceptual and mathematical relations contained in physical and physical-mathematical models (Lemmer, 2013; Zwickl, Hu, Finkelstein & Lewandowski, 2015). The importance of experimentation as foundational to understanding formal physics is supported by Mäntylä and Hämäläinen (2015), who have designed and evaluated an “experimental mathematization” sequence that progresses from qualitative experiments to quantifying experiments on electric circuits. Glazer (2011) argues that physics instructors should explicitly guide students in the integration of mathematics and physics knowledge (mathematization) because it generally does not happen by itself.

Theoretical Framework

Students show graph comprehension if they can read a graph, understand what it represents and can make meaningful deductions from it (Friel et al., 2001; Glazer, 2011). Graph comprehension and graph sense help students acquire graphical thinking skills, comprehend the nature of graphs (Friel et al., 2001) and learn to attach domain-specific meaning to variables and relations (Leinhardt et al., 1990). Students’ understanding of graphs is determined by their familiarity with the characteristics and content of a graph, their graph skills and their expectations (Glazer, 2011; Shah & Freedman, 2011). These aspects aid students in determining what inference to make and how to do it.

In a review article on statistical graphs Friel et al. (2001) indicated that students’ graph comprehension is influenced by the characteristics of the reader of the graph, and the context in which the graph is set, as well as visual decoding and judgement of the graph. Reader characteristics that advance graph comprehension include cognitive ability, graph experience, practical applications of graphs and general intelligence (Friel et al., 2001).

Context deals with the understanding of graphs in the situation in which it is set. Friel et al. (2001) looked at the influence of real-world contexts on graph comprehension. In physics data are often collected from real-world situations and the graph reader should be able to describe, represent, analyse and interpret the data within the relevant contextual frame. The interpretation of real-world graphs implies that relevant inferences are made and contextual problems are solved (Freedman & Shah, 2002). In the interpretation of the results obtained in the study reported here, the aspect of context as seen by Friel et al. (2001) was broadened to include relevant contextual knowledge and skills associated with graphs in mathematics and kinematics. Students’ contextual knowledge in mathematics and physics has a significant influence on their performance in science (Chambers & Andre, 1997; Hazari, Tai & Sadler, 2007) and determines how they interpret graphs (Shah & Freedman, 2011).

Visual decoding entails the identification of the visual features (e.g. the form) of the graph (Shah & Hoeffner, 2002), decoding information from the graph that has been encoded into the graph. Visual perception is thus used to interpret a graph (Legge, Gu & Luebker, 1989). Judgement occurs when the visual features of a graph are related to conceptual interpretation of the relations represented by the features (Shah & Hoeffner, 2002). Graph interpretations may focus on one data point (visual decoding) or it may require integration of information across data points (judgement) (Friel et al., 2001). The latter inferential process may involve computations, comparisons and identification of trends. Since students’ visual decoding of a graph affects the inferred conceptual relations and thus their judgement of the graph, students seem to find questions on comparison and trends more difficult than point-reading. They also have more success with graphs depicting distance or position than velocity or acceleration, which involves changes in variables (Shah & Hoeffner, 2002).

Aim and Research Questions

The aim of the research was to investigate factors that influence physics students' graph comprehension in kinematics. The following research question was addressed in the empirical study: how do reader characteristics, contextual knowledge and inferential processes influence physics students' kinematics graph comprehension with regard to the operations of reading coordinates, determining gradient and area under graphs, and connecting representations?

Research Method

The participants included 152 willing first-year physics students enrolled at the Central University of Technology in Bloemfontein, South Africa. All participants have studied physics and mathematics as separate subjects in secondary schools. Biographic details of the participants are discussed as part of reader characteristics in the results.

The research question was addressed with the aid of a questionnaire (quantitative data) followed up by interviews (qualitative data). The questionnaire (Phage, 2015) consisted of two sections of multiple choice questions, namely one on kinematics (30 items) and one on corresponding mathematics graph operations (27 items). The kinematics items were selected from the questionnaire of Beichner (1994) to assess the participants' understanding of using different graph operations on the level of the final South African school year. Only motion in one dimension was considered. The researchers added additional questions to probe deeper into students' understanding of the graph operations. The questionnaire contained four graph operations: reading of coordinates, comparing and calculating area under graphs and the gradients of graphs, and relating representations (algebraic equations and shape of graphs). Mathematics questions on linear functions that require execution of the same operations as the kinematics questions were then compiled.

The questionnaire was content validated by two academics in the research field and piloted with 30 physics students enrolled at the institution the previous year. The necessary changes were made. The reliability of the final questionnaire (Phage, 2015) was confirmed by Cronbach's alpha coefficients of 0.69 for the section on kinematics and 0.75 for the mathematics section.

The influence of reader characteristics on participants' performances in the mathematics and kinematics sections of the questionnaire were statistically determined using effect sizes for differences in means of independent groups (Ellis & Steyn, 2003). A value of $d = 0.2$ shows a small effect, $d = 0.5$ a medium, while $d \geq 0.8$ indicates a difference of practical significance. The participants' gender, their study courses and whether they completed school the previous year or before (i.e. whether a gap existed between their final school year and introductory university physics) were considered. The differences in average percentages in the mathematics and kinematics sections were statistically compared.

The results of the questionnaire were statistically analysed using effect sizes, because available sampling was used. Comparisons of the differences in proportions between mathematics and kinematics successes were interpreted according to Cohen's effect sizes using the McNemar test (Stokes, Davis & Koch, 2012). The effect size values indicate whether there is a practically significant difference between the proportions of participants who succeeded in answering the mathematics items correctly versus the proportion of participants who succeeded to doing so in the kinematics items. A small effect is indicated by $w = 0.1$, a medium effect by $w = 0.3$ while $w \geq 0.5$ indicates a practically significant difference between the two aspects considered. In this study the effect sizes were determined for pairs of mathematics and kinematics items requiring the same operation. A small effect size ($w < 0.3$) implies that participants performed similarly in a pair of items. In other words, they tended to answer either both the items correctly or both incorrectly. Large effect sizes ($w \geq 0.5$) show a practical significant effect, indicating a discrepancy in participants' responses to the two items. In such cases, when participants performed much better in the mathematics item than the relevant physics item, one can infer that they did not apply the mathematics knowledge they possessed in the kinematics item.

After completion of the analysis of the questionnaire, 14 willing participants of varying abilities were purposively selected and interviewed for further investigation of the effect of their kinematics context knowledge on their graph comprehension. The results of these structured interviews centred on their questionnaire responses were documented and used to explain trends in responses to the questionnaire. All ethical requirements of the institution were adhered to in all aspects of the research study.

Results of Empirical Study

Reader Characteristics

The number of participants and the average percentages obtained by each group are given in Table 1 for gender, Table 2 for the last year at school and Table 3 for the faculty in which they enrolled.

The results of Tables 1–3 show that small to medium variations in performances occurred due to differences in reader characteristics. With regard to gender, male participants outperformed female participants in both the mathematics and kinematics sections with medium effect size (≥0.5). Participants who had completed school the previous year did better in mathematics (effect size 0.43) than those who had had a gap of one or more years since their previous studies of mathematics, although only a small effect size (0.23) occurred for kinematics. An interesting result is the irrelevance of the faculty participants were enrolled in. Although admission requirements differed, engineering participants performed similarly to participants from the humanities and from the health and environmental sciences faculties.

Table 1. Comparison of the effect of reader characteristics on responses to the mathematics and kinematics sections: gender

Gender	% of participants	Mathematics	Kinematics
Male	53	67%	38%
Female	47	58%	32%
Effect sizes (*d*-value):		0.58	0.52

Table 2. Comparison of the effect of reader characteristics on responses to the mathematics and kinematics sections: last school year

Last school year	% of participants	Mathematics	Kinematics
Previous year	51	66%	36%
Earlier	49	59%	33%
Effect sizes (*d*-value):		0	0.23

Table 3. Comparison of the effect of reader characteristics on responses to the mathematics and kinematics sections: faculty

Faculty of study	% of participants	Mathematics	Kinematics
Humanities	23	61%	35%
Engineering and Information Technology	24	64%	35%
Health and Environmental Sciences	53	63%	35%
Effect sizes between pairs of faculties:		≤0.24	≤0.07

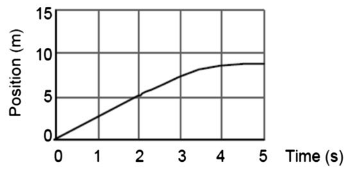
Contextual Knowledge and Inferential Processes

In the questionnaire as a whole as well as in each construct, the participants performed better in the mathematics items than the kinematics items requiring similar operations. The average for all mathematics items was 63% (standard deviation 16%), while the average mark in kinematics was only 35% (standard deviation 13%).

Reading coordinates

Overall participants performed well when reading coordinates, for example in kinematics item P6.1 (92%) shown in Figure 1. Similar good performances (>80%) were obtained in the other kinematics items as well as the two mathematics items that required simple reading of coordinates. According to the McNemar test no significant differences ($w \leq 0.2$) occurred between performances on coordinate reading in the kinematics and mathematics items.

P6. The position–time graph below shows the straight line motion of an object. Answer the following questions:



P6.1 The position at the 2 second point in the position–time graph is most nearly:	P6.2 The velocity at the 2 second point in the position–time graph is most nearly:
1. 0.4 m	1 0.4 m/s
2. 2.0 m	2 2.0 m/s
3. 2.5 m	3 2.5 m/s
4. 5.0 m	4 5.0 m/s
5. 9.0 m	5 10.0 m/s

Figure 1. An example of items testing for coordinate reading and gradient of kinematics graphs

Determination and interpretation of gradient

The position–time graph of Figure 1 was further used to determine whether students can infer from it the velocity at an instant in time (P6.2). Some 65% of the participants chose the correct option 3. It is interesting to note that nearly a fifth of the participants (20%) simply read off the value 5 from the position axis (option 4). The participants' difficulty to connecting the gradient of the position–time graph to velocity was confirmed in the interviews.

In order to investigate whether the participants exhibited the mathematical skill of determining the gradient at a point, the kinematics graph of Figure 1 was also given in the mathematics section using simply y and x variables instead of position and time. The question "What is the gradient of the graph when $x = 2$?" was answered correctly by 83% of the participants. This pair of corresponding kinematics and mathematics items was answered differently with medium effect ($w = 0.3$). A large percentage (28%) of participants who succeeded in the mathematics item could not answer the kinematics problem. The interviews further showed deficiencies in participants' understanding of the basic kinematics concepts of displacement, average and instantaneous velocity as well as acceleration. A common error was to equate average velocity to the ratio of position and time ($v = s/t$) instead of the ratio of change in position and time ($v = \Delta s/\Delta t$). This error was also evident from the interviews as well as other items in the questionnaire (e.g. P6.4 calculating velocity at 5 seconds from the graph in Figure 1), especially in cases where the gradient of kinematic graphs had to be determined for sections of line graphs that do not pass through the origin, i.e. in cases where $\Delta s/\Delta t \neq s/t$. Only 8% of participants had P6.4 correct and 17% successfully answered the corresponding mathematics item with a small difference ($w = 0.2$) in proportional answering.

Area under a graph

Figure 2 gives two of the questionnaire items that compared students' mathematical ability to determine area under a graph (item M7.2) and implement it in a kinematics graph (item P1).

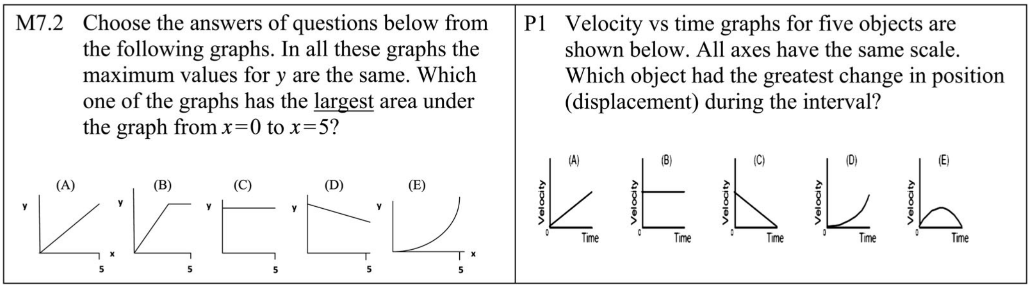


Figure 2. An example of corresponding mathematics and kinematics items testing for comparison of area under graph lines

The correct answer to the mathematics item M7.2 is graph (C), whereas graph (B) with the corresponding form is the correct option to item P1. While 69% of participants had the mathematics item correct, only 28% had the kinematics item correct. The practical significance of this difference in their responses is given by the large *w*-value of 0.63. This tendency also occurred in the other corresponding pairs of items on calculating the area under graphs. All *w*-values showed medium to large effects.

Relating representations

Corresponding mathematics and kinematics items (M4 and P20) on connection of graphic and algebraic representations show five graphs of different forms (Figure 3). The participants had to match a straight line (items M4.1 and P20.1), hyperbolic (M4.2 and P20.3) and quadratic (M4.3 and P20.2) mathematical function or kinematics equation to one of the given graphs.

The results show that 67 and 65% of participants recognised straight lines represented by the given expressions in M4.1 (graph (A)) & P20.1 (graph (A)) respectively. In the following mathematics items a similar percentage of participants associated graph (C) with the given quadratic function and recognized graph (E) to be hyperbolic. In the kinematics item P20.2, only 38% of participants associated

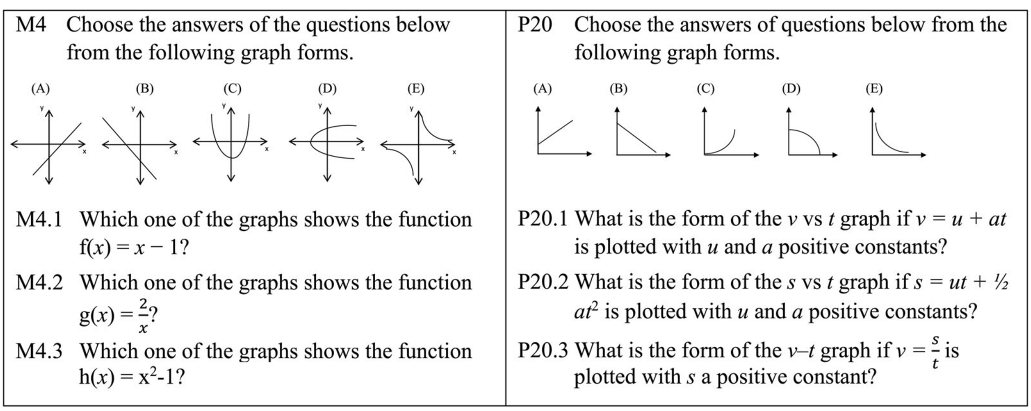


Figure 3. An example of corresponding items set in mathematics and kinematics contexts testing for relating graphical and function representations of graphs

the quadratic equation with graph (C) and a mere 27% recognized the hyperbolic form of the kinematics expression in P20.3 and correctly chose option (E). While a small *w*-value of 0.03 was obtained for the corresponding straight line graphs, the hyperbolic and parabolic equations yielded large *w*-values (0.58 and 0.44), implying medium to practically significant differences in responses in the two contexts. Participants' difficulties in connecting different representations of kinematics functions were confirmed in the interviews.

Discussion of Results

The results revealed that the majority of participants were successful in answering the mathematics questions on reading coordinates, area operations and relating representations. These participants showed conceptual understanding and thus effectively performed the inferential processes (visual decoding and judgement) in these graph operations. However, the participants generally struggled with the gradient operations, seemingly owing to a lack of conceptual understanding (contextual knowledge) of the mathematical concept of gradient.

In kinematics, the majority of participants performed weaker than in corresponding mathematical graph operations. Average percentages were <50%, except on coordinate reading, algebraic representations of straight line graphs and some gradient operations. It can thus be deduced that the contextual knowledge of the context of kinematics had an observable influence on the participants' kinematics graph comprehension.

Mathematics contextual knowledge required for visual decoding when reading of coordinates from a given graph entails the understanding that each point on the graph presents a specific *x*- and *y*-coordinate pair in the Cartesian coordinate system. In kinematics visual decoding can thus be done by simply substituting the kinematics variables (e.g. position (*m*) and time (*s*) in P6.1, Figure 1) for the *x*- and *y*-coordinates on the mathematical Cartesian system without scientific comprehension of the kinematics concepts. The participants answered corresponding mathematics and kinematics items with no significant differences. This indicates that the participants transferred their ability to read coordinates and do visual decoding in mathematics to kinematics.

Determining the velocity from a position–time graph (P6.2, Figure 1) requires context knowledge and understanding of velocity as the rate of change of the position of a moving object that is presented by the gradient at a point on the position-time graph. The participants calculating the correct velocity performed the correct visual decoding and judgement. Those reading off the position instead of determining the gradient performed only visual decoding. This problem corresponds to the so-called height–gradient confusion reported by McDermott et al. (1987), Beichner (1994) and Christensen and Thompson (2012) and may possibly be ascribed to participants' familiarity with reading coordinate values that they use when they do not have the kinematics contextual knowledge that the gradient has to be determined. The corresponding mathematics items calculating the gradient were answered differently with medium effect, indicating that the participants who had the contextual knowledge that velocity is presented by the gradient of a position–time graph transferred their ability to determine gradient in mathematics to kinematics.

Although mathematics questions regarding the gradient of a straight line starting at the origin were generally well answered, some of the questions revealed deficiencies in conceptual understanding of gradient, namely that gradient is the ratio of the *change* in *y*-values to the *change* in *x*-values ($\Delta y/\Delta x$) and not the ratio between variables (y/x). These deficiencies were also applied in the kinematics context, e.g. determining average velocity as $v = s/t$ instead of $v = \Delta s/\Delta t$ transferring their incorrect understanding of the mathematics concept of gradient to kinematics. The lower percentages generally obtained in the kinematics questions in this construct further showed that participants' judgement whether to determine a function value or gradient were hindered by weak contextual knowledge of kinematics concepts, causing some participants not to transfer their mathematical skill of gradient to kinematics.

The results on area operations indicate that the majority of participants had mathematical contextual knowledge about areas of geometric forms and could execute visual decoding and judgement. In the corresponding kinematics questions the participants firstly had to take the kinematics context of the questions into account before deciding what visual decoding and judgement to do. The poor performance of the participants in the kinematics questions indicated that they encountered problems with the context of this graphics operation, indicating deficiencies in kinematics contextual knowledge, as reported by Beichner (1994), McDermot et al. (1987) and Palmquist (2001). The participants' lack of contextual knowledge in kinematics hindered the transfer of their mathematical skill on area to kinematics.

When relating representations, the majority of participants seem to be familiar with the standard mathematical format of the equations, successfully decoded the mathematical graphs visually and connected the equations with the corresponding graphical representations. Visual differences of the given kinematics hyperbolic and parabolic graphs as compared with the more familiar mathematical forms could have contributed to the lower performances by participants in kinematics. A contributing factor to these differences could be the context of kinematics' requirement that time is always positive. Therefore, only the parts of the graphs in the first quadrant as well as the form of the equations are visually similar in the two contexts, causing participants' uncertainties in visual decoding in the kinematics questions. It is also possible that the participants did not recognize the kinematics expressions as having the same format as the mathematical functions with displacement or velocity as the dependent variable presented on the vertical axis and time as the independent variable on the horizontal axis, since these correspondences are generally not pointed out in South African school textbooks. It could also be possible that the participants found it difficult to identify time as the variable confusing it with other parameters, e.g. u and a. In physics, expressions consist of many unknowns of which some are considered as parameters and other as variables. The graph reader has to know which two unknowns to identify as variables. Consequently, here too the participants were only able to transfer the mathematics knowledge of straight-line graphs to kinematics, and not the other forms, owing to a lack of kinematics contextual knowledge.

In summary, in both mathematics and kinematics the participants did not have visual decoding problems when reading coordinates. In the other graph operations visual decoding and judgement were based on contextual knowledge. Doing visual decoding only by reading off a value from the graph, instead of performing judgement too by determining the gradient at a specific point on a graph (height–gradient confusion) is due to insufficient contextual knowledge on the meaning of gradient in the kinematics context. The lack of contextual knowledge in kinematics also hampered the participants' transfer of mathematics skills to kinematics graphs with regard to area and representations of parabolic and hyperbolic relations, and to a lesser extent in the case of gradient. This contributed to differences in participants' performance when corresponding mathematics and kinematics graph operations were required. This study also pointed out that, in the case of sufficient kinematics contextual knowledge, but a lack in mathematics conceptual knowledge on gradient, the mathematical misunderstanding is transferred to kinematics, causing incorrect judgement of the operation to be performed. The results of the study emphasises that, without knowing that they are required to determine or compare gradients (or areas) of kinematics graphs, students cannot answer the questions with confidence even though they may possess the necessary mathematics knowledge.

Conclusions

This comprehensive study of a variety of graph operations showed that students' graph comprehension is more complex than indicated by studies of only one graph operation or one factor of graph comprehension. Attending to the research question, all of the factors investigated influenced students' graph comprehension, although in different ways and to different extents. The reader characteristics of the participants played a lesser role in participants' graph comprehension in kinematics, but cannot be neglected since they influence their contextual knowledge and effective learning of new knowledge. The results stress the influence of the context in which the graph is set. Contextual knowledge encompasses content knowledge of mathematics and kinematics on the conceptual level as well as knowledge of aspects regarding the nature of the subject. Participants' contextual knowledge of variables and their relations in physical systems influence their ability to judge what visual aspects to decode and inferences to make from kinematics graphs. The context of the graph also determines its visual features, for example that time only has positive values. Therefore mathematics and physics questions requiring the same graph operations may have different appearances, which retard transfer of existing mathematics knowledge to kinematics. The inferential processes (visual decoding and judgement) to be performed thus depend on both the context and the operations required by the question and students' knowledge of them.

A necessity for students' comprehension of physical-mathematical graph models is thus a deep foundation of relevant conceptual physics and mathematical knowledge that is embedded in the real-world and presented according to conventions. Knowledge about the physical system and how it is represented by a given graph aids in students' visual decoding and judgement of how to answer the question and perform the relevant operations.

Limitations and Recommendations

Although the specific findings of this study concerning the four graph operations, reading coordinates, determining the gradient of and the area under a line graph and connecting representations, in the kinematics context, may be different for another group of first year physics students or another physics context; all of the factors may affect graph comprehension and should thus be taken into account. The influence of contextual knowledge on visual decoding, judgement and transfer of skills from mathematics to other contexts are perceived to be applicable in general. In order to perform successfully in graphs representing a physical-mathematical model, students have to know and understand the underlying mathematics and physical systems (i.e. concepts, principles, models and assumptions) and how to blend these effectively by taking the context into account. Deficiencies in any one of these aspects should prevent success in interpreting kinematics graphs.

Students should explicitly be guided in progression towards physics graph comprehension. Engagement in practical experiences of collection and analysis of real data (Woolnough, 2000; Lemmer, 2013) is strongly recommended. Real-life situations and physics laboratory experiences provide opportunities to work with variables and gain comprehension of their relative changes. Relations between the measured variables can then be meaningfully expressed mathematically and visualised in graphs as physical-mathematical model. Future research should be done on the design and evaluation of teaching sequences aimed to construct and elaborate (refine) students' knowledge and comprehension of kinematics graphs.

Disclosure Statement

No potential conflict of interest was reported by the authors.

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References

Beichner, R.J. (1994). Testing student interpretation of kinematics graphs. *American Journal of Physics 62*(8), 750–762.

Chambers, S.K., & Andre, T. (1997). Gender, prior knowledge, interest, and experience in electricity and conceptual change manipulations in learning about direct current. *Journal of Research in Science Teaching 34*, 107–123.

Christensen, W.M., & Thompson, J.R. (2012). Investigating graphical representations of slope and derivative without a physics context. *Physical Review Special Topics—Physics Education Research 8*, 023101.

Ellis, S.M., & Steyn, H.S. (2003). Practical significance (effect sizes) versus or in combination with statistical significance (*p*-values). *Management Dynamics 12*, 51–53.

Freedman, E.G., & Shah, P. (2002). Toward a model of knowledge-based graph comprehension. In M. Hegarty, B. Meyer, & N.H. Narayanan, N.H. (Eds.), *Second international conference diagrams 2002* (pp. 18–20). Heidelberg: Springer.

Freitas, I.M., Jiménez, R., Mellado, V. (2004). Solving physics problems: The conceptions and practice of an experienced teacher and an inexperienced teacher. *Research in Science Education 34*, 113–133.

Friel, S.N., Curcio, F.R., & Bright, G.W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. *Journal for Research in Mathematics Education 32*, 124–158.

Glazer, N. (2011) Challenges with graph interpretation: a review of the literature. *Studies in Science Education 47* (2), 183–210.

Hazari, Z., Tai, R.H., & Sadler, P.M. (2007). Gender differences in introductory university physics performance: The influence of high school physics preparation and affective factors. *Science Education 91*, 847–876.

Lemmer, M. (2013). Nature, cause and effect of students' intuitive conceptions regarding changes in velocity. *International Journal of Science Education 35*, 239–261.

Lemmer, M., & Gunstone, R. (2016). Physical models: A crucial link between reality and mathematical models. Paper presented at SAIP Conference, University of Cape Town, South Africa.

Legge, G.E., Gu, Y., & Luebker, A. (1989). Efficiency of graphical perception. *Perception and Psychophysics 46*, 365–374.

Leinhardt, G., Zaslavsky, O., & Stein, M.K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research 60*, 1–64.

Linder, A., Airey, J., Mayaba, N., & Webb, P. (2014). Fostering disciplinary literacy? South African physics lecturers' educational responses to their students' lack of representational competence. *African Journal of Research in Mathematics, Science and Technology Education 18*(3), 242–252.

Mäntylä, T., & Hämäläinen, A. (2015). Obtaining laws through quantifying experiments: Justifications of pre-service physics teachers in the case of electric current, voltage and resistance. *Science and Education 24*, 699–723.

McDermott, L.C., Rosenquist, M.L., & Van Zee, E.H. (1987). Student difficulties in connecting graphs and physics: Examples from kinematics. *American Journal of Physics 55*(6), 503–513.

Meredith, D.C., & Marrongelle, K.A. (2008). How students use mathematical resources in an electrostatics context. *American Journal of Physics 76*, 570–578.

Palmquist, B.C. (2001). Assessing motion graphs. *The Science Teacher 68*, 75–77.

Phage, I.B. (2015). An analysis of students' knowledge of graphs in mathematics and kinematics. Unpublished MSc dissertation, North-West University, South Africa.

Potgieter, M., Harding, A., & Engelbrecht, J. (2008). Transfer of algebraic and graphical thinking between mathematics and chemistry. *Journal of Research in Science Teaching 45*, 197–218.

Redish, E.F. (2005). Problem solving and the use of math in physics courses. In *Proceedings of the conference on world view on physics education: Focusing on change*, New Delhi.

Redish, E.F., & Kuo, E. (2015). Language of physics, language of math: Disciplinary culture and dynamic epistemology. *Science & Education 24*, 561–590.

Scott, F. (2012). Is maths to blame? An investigation into high school students' difficulty in performing calculations in chemistry. *Chemistry Education Research and Practice 13*, 330–336.

Shah, P., & Freedman, E.G. (2011). Bar and line graph comprehension: An interaction of top-down and bottom-up processes. *Topics in Cognitive Science 3*, 560–578.

Shah, P., & Hoeffner, J. (2002). Review of graph comprehension research: Implications for instruction. *Educational Psychology Review 14*, 47–69.

Shaughnessy, J.M., Garfield, J., & Greer, B. (1996). This is a chapter In A.J. Bishop, M.A. Clements, C. Keitel-Kreidt, J. Kilpatrick & C. Laborde (Eds.), *International handbook of mathematics education: Data handling* (pp. 205–237). Dordrecht: Springer.

Stokes, M.E., Davis, C.S., & Koch, G.G. (2012). *Categorical data analysis using the SAS system*. Cary, NC: SAS Institute.

Tuminaro, J., & Redish, E.F. (2004). Understanding students' poor performance on mathematical problem solving in physics. In J. Marx, S. Franklin & K. Cummings (Eds.), *American Institute of Physics: 2003 Physics education research conference* (pp. 113–116).

Uhden, O., Karam, R., Pietrocola, M., & Pospiech, G. (2012). Modelling mathematical reasoning in physics education. *Science & Education* 21, 485–506.

Woolnough, J. (2000). How do students learn to apply their mathematical knowledge to interpret graphs in physics? *Research in Science Education 30*, 259–267.

Zwickl, B.M., Hu, D., Finkelstein, N., & Lewandowski, H.J. (2015). Model-based reasoning in the physics laboratory: Framework and initial results. *Physical Review Special Topics—Physics Education Research 11*, 020113.