Using Eye-Tracking to Assess the Application of Divisibility Rules when Dividing a Multi-Digit Dividend by a Single Digit Divisor

Pieter Potgieter
Central University of Technology, Free State
South Africa
pieter@cut.ac.za

Pieter Blignaut
University of the Free State
South Africa
BlignautPJ@ufs.ac.za

ABSTRACT

The Department of Basic Education in South Africa has identified certain problem areas in Mathematics of which the factorisation of numbers was specifically identified as a problem area for Grade 9 learners. The building blocks for factorisation should already have been established in Grades 4, 5 and 6. Knowing the divisibility rules, will assist learners to simplify mathematical calculations such as factorisation of numbers, manipulating fractions and determining if a given number is a prime number. When a learner has to indicate, by only giving the answer, if a dividend is divisible by a certain single digit divisor, the teacher has no insight in the learner's reasoning. If the answer is correct, the teacher does not know if the learner guessed the answer or applied the divisibility rule correctly or incorrectly.

A pre-post experiment design was used to investigate the effect of revision on the difference in gaze behaviour of learners before and after revision of divisibility rules. The gaze behaviour was analysed before they respond to a question on divisibility.

It is suggested that if teachers have access to learners' answers, motivations and gaze behaviour, they can identify if learners (i) guessed the answers, (ii) applied the divisibility rules correctly, (iii) applied the divisibility rules correctly but made mental calculation errors, or (iv) applied the divisibility rules wrongly.

CCS CONCEPTS

• **Computing methodologies** → Tracking; Activity recognition and understanding • **Information systems** → Relevance assessment • **Social and professional topics** → Student assessment

KEYWORDS

Divisibility rules, eye-tracking, Mathematics

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1 INTRODUCTION

In 2013 and 2014, around 7 million learners of South Africa from Grade 1 to Grade 6 and Grade 9 took part in the Annual National Assessments (ANA) [1, 2] Only 3% of Grade 9 learners achieved more than 50% in Mathematics. The Department of Basic Education [3, 4] has, through the ANAs, identified certain problem areas of which the factorisation of numbers was specifically identified as a problem area for Grade 9 learners. It is expected that the building blocks for factorisation should already have been established in Grades 4, 5 and 6 and related questions appeared in the ANA papers for these grades.

Knowing the divisibility rules (Table 1) would enable learners to quickly determine if a number, referred to as the dividend, is divisible by a specific single digit divisor without having to do long calculations. Evidence of formal instructions to apply divisibility rules for divisors 2 to 12 was found in the workbook for Grade 5 to Grade 7 learners [5-7], but despite this early exposure, the fact that Grade 9 learners lack the ability to do factorisation, is concerning.

Table 1: Divisibility rules

Divisor	Rule
2	The last digit must be even.
3	The sum of the digits must be divisible by 3.
4	The number formed by the last two digits must be divisible by 4.
5	The last digit must be 0 or 5.
6	The number must be divisible by both 2 and 3 according to the above-mentioned rules.
8	The number formed by the last three digits must be divisible by 8.
9	The sum of all the digits must be divisible by 9.

The most basic formats of assessment consist of multiple choice and true/false questions [8]. When a learner has to indicate if a dividend is divisible by a certain single digit divisor by only answering true or false, the teacher has no insight in the learner's reasoning. If the answer is correct, the teacher does not know if the learner guessed the answer or applied the divisibility rule correctly.

This study makes use of eye-tracking technology to observe learners' gaze behaviour while they are applying the divisibility rules. The research question that will be addressed is if it is possible that learners' gaze behaviour can indicate whether they applied the divisibility rules correctly when they correctly indicated if a dividend is divisible by a specific single digit divisor.

The rationale of the study will be provided in the following section. Thereafter, previous attempts where eye-tracking was used to analyse gaze behaviour while participants solve mathematical problems, will be discussed. This will be followed by the experimental details. The results will refer to the the effect of grade, revision and divisor on the percentage of fixation time per digit as well as the gaze behaviour of learners who provide incorrect answers. The minimum attention required per digit per divisor will be investigated and some conclusions will be made.

2 RATIONALE OF THE STUDY

Performing division can be time consuming and complex. If, however, the quotients or remainders are not required, divisibility rules can be used to determine if one integer is divisible by another [9]. "***a*** divides ***b*** if and only if ***a*** is a factor of ***b***. When ***a*** divides ***b***, we can also say that ***a*** is a divisor of ***b***, ***a*** is a factor of ***b***, ***b*** is a multiple of ***a***, and ***b*** is divisible by ***a***" [10]. Divisibility rules present a short way of determining whether ***a*** divides ***b*** without actually performing the division.

2.1 Basic background

Rules can be used for problem solving, but it must be applied with understanding [11]. The concepts of division and divisibility are important factors that help learners move from Arithmetic to Algebra [12].

Knowledge of the divisibility rules can make life easier for learners in various application areas. Some of these areas include finding factors of a dividend, determining if a dividend is a prime number, calculating factor pairs of a dividend, simplifying fractions and calculating the lowest common multiple and least common denominator of two dividends.

2.2 Teaching

The teachers' role is to transfer knowledge and mathematical rules to learners [13] and provide guidance that encourage learners to express themselves and improve their reasoning skills [14-16]. Teachers can determine if learners grasped certain concepts and are in a position to decide when to intervene if learners struggle with specific concepts [17]. Learners' understandings of mathematical concepts improve when they recognised their misunderstandings or mistakes, or when the teacher point out their mistakes or misunderstandings [18].

2.3 Problem solving

There are different types of knowledge that learners could use to solve a mathematical problem. The most common types of knowledge are conceptual and procedural knowledge. Procedural knowledge can be classified as a series of steps, or actions, done to accomplish a goal [19]. When learners use rules to solve mathematical problems, it can be classified as procedural knowledge [20]. The action to determine divisibility without actually doing it, is a type of procedural activity and procedural understanding [9].

2.4 True/false questions

The probability of guessed answers for true/false questions is high [21, 22]. The credibility of true/false questions can be improved by asking learners to provide reasons for their answers [8]. However, when learners are expected to provide a reason for an answer, they may have difficulty in expressing how they reached a specific answer [23]. Each class has its "Lucky Larry", who manages to get the correct answer from incorrect reasoning [24]. Learners want to provide satisfactory answers to questions and they will often act as if they know the correct strategy – although there is no evidence that they are aware of it [25]. In other words, learners may seem confident about their reasoning even though the reasoning is incorrect [26, 27]. The use of certain strategies indicates the understanding of mathematical concepts and learners should have the ability to recognise where and when specific strategies could be used to simplify processing of a problem [28]. Teachers cannot identify the incorrect application of a divisibility rule if the correct answer and motivation were provided.

3 GAZE BEHAVIOUR

Eye movements reveal detailed information on the procedure of how a learner approaches a problem and how the learner reaches the solution to the problem [29]. Since most eye-tracking systems make use of infrared illumination that is invisible to the eye and does not distract or annoy the user [30], eye-tracking can be used as a non-intrusive technology to investigate learners' gaze behaviour [31] while they are doing Mathematics.

3.1 Fixations

A fixation is defined as the maintaining of the visual gaze on a single location, and a saccade is a quick motion of the eye from one fixation to another [32]. The duration of a fixation is at least 100 to 150 milliseconds and a fixation represents the gaze on an attention area [30]. As such, eye-tracking can be used to inspect participants' distribution of visual attention while they are solving problems [29]. Eye-tracking can thus also reveal which areas participants are not inspecting [33]. The correct patterns of gaze behaviour could be associated with learners who possess solid factual knowledge of a problem [34]. Analysis of gaze behaviour can be used to identify the academic potential and cognitive activities of learners [31, 35-37]. There is also huge potential in investigating how complex numerical tasks are performed and how the strategies of well-performing learners differ from those of poor-performing learners [38]. Participants who know how to perform a task, spend more time on relevant areas than on irrelevant areas [39] and shift their gaze and attention after practice to more appropriate areas of interest [40].

3.2 Eye-tracking complements verbal responses

Gaze behaviour cannot be obtained from verbal responses [41] and the verbal explanation of reasoning can on its own not be used to derive which strategies were used [33]. Verbal responses, together with eye movement data, can be used to examine cognitive processes [41]. Learners' eye movements can potentially provide more information than traditional written assessments and also reveal the strategies that learners use when dealing with mathematical problem solving [39]. In addition, learners' gaze behaviour might also be used to investigate if there were implicit forms of understanding that accompany incorrect verbal responses [42]. During the capturing of eye movements, participants could be asked additional questions in order to supplement the data [43].

3.3 Peripheral vision

A specific limitation of eye-tracking, known as the dissociation problem, is that participants may fixate on a certain area while they are paying attention to another area [43]. Visual attention may be overt or covert [44]. Participants may use peripheral vision to inspect areas of interests that they do not directly fixate on [23]. Information within an AOI, which were observed by peripheral vision or direct fixations, can be absorbed by the short-term memory if it stays there long enough to be recalled [45].

There are limitations when eye-tracking is used to investigate learners' reasoning while they are completing mathematical assessments, because one can only determine where they are looking and not what they are thinking at any specific moment [43]. Although gaze behaviour is indicative of cognitive processes, it will not be fully known what the brain absorbs while the participant is fixating on an object [43]. Eye-tracking is not an alternative for any other method of assessment, but it could be used in combination with other methods, such as verbal responses, to analyse thought processes [17, 43, 46].

3.4 Attempts to identify gaze behaviour in mathematical problem solving

Although many eye-tracking studies have already been performed on graphs, formulas, geometry, etc. [25, 32, 47-52], the discussions in this section will focus on the use of eye-tracking in applicable mathematical areas relating to the divisibility rules.

Godau [53] and Godau et al. [28] conducted a study to determine the ability of Grade 1 to Grade 3 learners to spontaneously notice commutativity (a+b+c = a+c+b) and it was concluded that learners used search processes to identify a suitable strategy to use and that better understanding of concepts led to an improvement in the strategy used and vice versa. There was no evidence that the order and number of fixations that participants moved their gaze had an effect on whether or not the correct strategy was used [54]. Previous research studies found that the magnitude of numbers determine fixation duration [44].

Cimen [55] performed a case study on division theory where (i) mathematical instruction was offered to elucidate the relationship between a divisor, dividend, quotient and remainder; and (ii) where participants were assessed on the learning material and it was found that participants built connections between related concepts of divisibility.

4 EXPERIMENTAL DETAILS

The general pre-post experimental design requires the same group of individuals to be measured prior to and after treatment [56]. This design was chosen for this study because there was no control group and the same learners participated before and after the divisibility rules were explained or revised.

4.1 Assessments

Seventy-eight learners from Grade 4 to Grade 7 participated in two assessments with eye-tracking. Each assessment focused on the ability of the learners to apply the divisibility rules on divisors 2 to 9, excluding 7. Divisor seven (7) was excluded because the rule for divisibility is more complicated than the rules for the other divisors in the range.

The first assessment was an unprepared test where learners had to verbalise their answers and also provide reasons while looking at a series of five-digit dividends one after the other on a computer screen. Revision was done on the divisibility rules a month after the first assessment. The second assessment was done a week after the revision lesson, in the same way as the first assessment. Only learners who participated in both assessments were used for the analysis.

Fourteen questions, two per divisor, were asked in random order in each of the two assessments. The same set of questions was used for both assessments. For each divisor there was a question where the dividend was divisible by the divisor and a question where the dividend was not divisible by the divisor.

A learner should be able to answer three true/false questions or three short-answer questions per minute [57] and therefore each dividend was displayed for 20 seconds. The researcher initiated the move to the next stimulus as soon as the learner provided an answer or after 20 seconds have expired. The researcher also recorded the learners' verbal responses.

4.2 Compilation of Dividends

All the dividends that were presented to learners were five-digit numbers as Grade 4 learners are supposed to identify a five-digit number [58].

During the pilot phase of the study, it was observed that learners inspected only the last digit or the number formed by the last two digits if they did not know the divisibility rule. Therefore, most of the dividends were compiled such that when learners inspected only the last digit or the number formed by the last two digits, the answer would probably be incorrect.

A calculator was not allowed and the choice of digits in the dividends was such that learners could do mental calculations. All dividends were chosen such that the sum of all digits varied from 18 to 20 (for example, 6+3+1+7+2=19).

4.3 Stimuli

The stimuli were presented as slides in landscape format and optimised for the laptop display with a resolution of 1600×900 pixels (36.09°×20.77°). "Digit 5" refers to the leftmost digit, while "Digit 1" refers to the rightmost digit. The A and B "digits" were placed at the sides of the dividend to minimise the positional advantage that the first and last digits would enjoy (Figure 1).

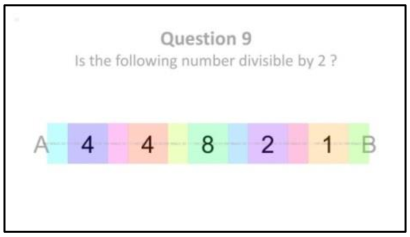

Figure 1: Areas of interest

The dividends, with spaces between the digits, were displayed in an Arial typeface with font size 72 (0.88°×1.32°). The dividend and all characters were spread out evenly across the display [59]. The distance between the digits of the dividend was 3.97°.

Each digit was included in an area of interest (AOI) of 160×160 pixels (3.26°×3.26°) (Figure 1). AOIs of 80×160 pixels (1.63°×3.26°) were inserted between the digits to capture fixations between digits. AOIs of 80×160 pixels were also placed at the ends of the dividend. The fixation time on the AOIs were used to calculate the percentage of total fixation time on the entire dividend for each one of the five digits. Fixations on the AOIs between digits contributed 50% to each of the digits on either side.

The instruction for each question was initially displayed without the dividend, for example "Is the following number divisible by 2?" Once the learner had time to read the instruction, the dividend was displayed. The instruction was still visible in case the learner forgot which divisor to use, but it was displayed in grey to minimise distraction from the dividend. As soon as a learner verbalised his/her answer ("yes" or "no"), the dividend was removed and the learner was requested to provide a reason for his/her answer.

4.4 Research instruments

A 60 Hz Tobii X2-60 eye-tracker was used to capture the gaze behaviour of learners. Tobii Studio (version 12) [60] was used to obtain the percentage of total fixation time on each AOI around the digits of the dividend for all the eye-tracking recordings. These percentages, together with learners' responses (answer and reason), were entered into an MS Excel spreadsheet to prepare the data for statistical analysis. A nine point calibration was done prior to the assessment to ensure good quality of the eye gaze recordings.

5 EFFECT OF GRADE, REVISION AND DIVISOR ON FIXATION TIME PER DIGIT

The effect of learner's grade, revision and divisor on the percentage of fixation time per digit are analysed below for learners who provided the correct answer and reason (A✓R✓). These learners probably applied the divisibility rules correctly and fixated on the required digits to conclude their answers. Learners who provided an incorrect reason (A✓R×) probably did not know the relevant divisibility rule and their fixations on the digits would not be reliable. Learners who provided a correct motivation but an incorrect answer (A×R✓) probably made mental calculation errors or applied the divisibility rule incorrectly and therefore their fixations on the digits could be misleading. Recordings where the quality of the eye-tracking data was such that there were no fixations on the areas of interest, were discarded.

Two questions were presented to learners for every divisor: one question where the dividend was divisible by the divisor and one question where it was not divisible. Using a within-subjects one-way analysis of variance, it was found that there is no significant (α = .05) difference in the percentage of learners who provided the correct answer and reason (A✓R✓) if a dividend is divisible by a divisor or not and therefore the influence of divisibility was discarded for further analysis.

5.1 Effect of Grade on Percentage of Fixation Time

The results of a within-subjects one-way analysis of variance (ANOVA) for the effect of the learner's grade on the percentage of fixation time (%) while controlling for the effect of revision, divisor and digit, were investigated. With the exception of four of the 70 cases (7 divisors, 5 digits, before and after revision), it was found that the effect of grade on the percentage of time that learners fixate on the respective digits, was not significant (α = .05).

5.2 Effect of Revision on Percentage of Fixation Time

5.2.1 Learners who performed better after revision. Learners who provided an incorrect reason for a question in the first assessment, irrespective of what their answers were (A✓R× or A×R×), did not know the relevant divisibility rule. If the same learners provided the correct answer and reason for the same question after revision (A✓R✓), they probably benefited from the revision session and would now be able to apply the divisibility rule.

Table 2 shows the results of a within-subjects repeated-measures ANOVA for the effect of revision on the percentage of fixation time while controlling for divisor and digit for learners who benefited from the revision (learners in A✓R× or A×R× before revision and A✓R✓ after revision). N indicates the number of responses (there were two questions per divisor, thus two responses per learner) in the respective groups.

No significant difference (α = .05) between the percentage fixation time on the A and B "digits" before and after revision was found for anyone of the divisors. There were also no significant differences (α = .05) for divisor 2. This could be due to the trend (Section 6.2) that if learners did not know the reason, they fixated mainly on the last two digits of the dividend which correlates with the divisibility rule of divisor 2. Although the same argument applies to divisor 5, a significant p value (α = .05) was found on digit 1 for divisor 5. Significant p values (α = .05) were found for all the digits for divisors 3, 6 and 9 except for divisor 3, digit 3. Significant p values (α = .05) were found for some of the digits for divisors 4 and 8. Therefore, it can be surmised that revision has a significant impact on the percentage of fixation time for learners who benefited from the revision.

5.2.2 Learners who provided the correct answer and reason before and after revision. If the same learners provided the correct answer and reason in Assessment 1 and again in Assessment 2, it probably means that they knew the divisibility rules before revision and that

Table 2: The effect of revision on the percentage of fixation time on a digit for learners who benefited from revision. N indicates the number of responses in the respective groups (A✓R× or A×R× before revision and A✓R✓ after revision)

Divisor	Digit	N	Percentage of total fixation time		Effect	p
			Before revision	After revision		
2	A&B	7	0.25		-	
	5		19.21		-	
	4		10.18	0.79	F(1, 6)=3.10	.129
	3		7.88	1.35	F(1, 6)=3.14	.127
	2		23.16	35.32	F(1, 6)=1.08	.339
	1		39.33	62.54	F(1, 6)=2.56	.161
3	A&B	70	0.23	0.07	F(1, 69)=2.59	.112
	5		1.99	13.28	F(1, 69)=48.01	**.000**
	4		6.49	27.51	F(1, 69)=119.24	**.000**
	3		18.35	22.18	F(1, 69)=3.71	.058
	2		44.17	24.17	F(1, 69)=68.25	**.000**
	1		28.78	12.73	F(1, 69)=30.68	**.000**
4	A&B	32	0.34	0.05	F(1, 31)=1.15	.292
	5		2.39	3.37	F(1, 31)=.21	.653
	4		3.28	2.33	F(1, 31)=.42	.520
	3		7.98	3.65	F(1, 31)=4.34	**.046**
	2		49.50	52.83	F(1, 31)=.33	.569
	1		36.52	37.76	F(1, 31)=.05	.819
5	A&B	3	0.04		-	
	5		6.97		-	
	4		14.68		-	
	3		14.80		-	
	2		40.63	8.88	F(1, 2)=2.41	.261
	1		22.89	91.12	F(1, 2)=163.08	**.006**
6	A&B	54	0.19	0.13	F(1, 53)=.37	.548
	5		2.77	10.59	F(1, 53)=38.93	**.000**
	4		3.33	16.08	F(1, 53)=42.83	**.000**
	3		13.87	20.13	F(1, 53)=5.74	**.020**
	2		48.11	31.66	F(1, 53)=26.10	**.000**
	1		31.74	21.42	F(1, 53)=7.79	**.007**
8	A&B	32	0.15	0.06	F(1, 31)=.56	.459
	5		6.32	0.78	F(1, 31)=6.45	**.016**
	4		8.04	2.81	F(1, 31)=5.29	**.028**
	3		19.29	29.91	F(1, 31)=7.10	**.012**
	2		45.40	51.30	F(1, 31)=1.46	.236
	1		20.79	15.14	F(1, 31)=2.79	.105
9	A&B	78	0.13	0.20	F(1, 77)=.38	.538
	5		3.44	10.95	F(1, 77)=37.28	**.000**
	4		6.78	15.98	F(1, 77)=33.21	**.000**
	3		16.51	22.79	F(1, 77)=10.11	**.002**
	2		44.06	29.92	F(1, 77)=34.81	**.000**
	1		29.07	20.17	F(1, 77)=10.16	**.002**

revision was actually unnecessary. The results of a within-subjects repeated-measures ANOVA for the effect of revision on the percentage of fixation time on the respective digits for learners in A✓R✓ at both occasions were investigated. As expected, the results show that learners who knew the divisibility rules before revision spent nearly the same percentage of fixation time on the different digits before and after revision. Significant p values (α = .01) were only found for digits 1, 4 and 5 for divisor 2. Therefore, it seems that revision does not have an influence on the percentage of time spent on a digit if learners knew the divisibility rule before revision.

5.3 Effect of Divisor on Percentage of Fixation Time

5.3.1 Answer and reason correct (A✓R✓). When learners provided a correct answer and reason, irrespective of whether it was before or after revision, one can infer that the learners knew the divisibility rule and also how to apply it. A series of one-way analyses of variance (ANOVA) showed a significant (α = .001) effect of divisor on the percentage of fixation time on each one of the five digits for both assessments for learners who provided the correct answer and reason. This result is no surprise as it is expected, for example, that learners have to focus on all digits for divisibility by 3 while they only need to inspect the last digit to determine if a dividend is divisible by 2 or by 5.

5.3.2 *More answer/reason combinations.* The previous analysis was performed for learners in A✓R✓. When learners provided an incorrect reason (R×), one can infer that the learners did not know the divisibility rule and guessed the answer. When learners provided an incorrect answer but a correct reason (A×R✓), it could mean that the learners knew the divisibility rule but probably made mental calculation errors or applied the rule incorrectly. This was confirmed when learners verbalised their calculations aloud. Examples of such verbalisations were: "75133 are divisible by 3 since 7+5+1+3+3 = 18" and "3 is a factor of 19".

Table 3 shows the average percentage of fixation time of learners in the different answer/reason combinations per digit and divisor. The maximum possible value of N is 312 (78 learners took part in 2 assessments × 2 questions per divisor). As mentioned before, recordings where the quality of the eye-tracking data was such that there were no fixations on the areas of interest were discarded. Table 3 shows that if a learner provided an incorrect answer and reason (A×R×), the most attention was on digit 1 and digit 2 – irrespective of the divisor. Learners who provided the correct answer but with a wrong explanation of how they arrived at their answers (A✓R×) also focused mainly on digit 1 and digit 2.

6 GAZE BEHAVIOUR OF LEARNERS WHO PROVIDE INCORRECT ANSWERS

6.1 The Effect of Correctness of Answer on the Percentage of Fixation Time per Digit (A×R✓ vs A✓R✓)

Learners who apply a divisibility rule correctly (R✓) should focus on the correct digits, irrespective of whether their answers are correct or not. A series of one-way ANOVAs (excluding divisor 2 and controlling for divisor and digit) confirmed that, with the exception of divisor 6, correctness of the answer had no significant (α = .01) effect on the percentage of fixation time per digit. Divisor 2 was not included because no participants applied the rule correctly along with an incorrect answer.

Table 3: Answer/reason combinations with percentage fixation time per digit. (Key: A=Answer; R=Reason; ×=Incorrect; ✓=Correct; N=number of responses)

		Divisor													
		2		3		4		5		6		8		9	
Combination	Digit	N	%	N	%	N	%	N	%	N	%	N	%	N	%
	A & B		0.32		0.09		0.15		0.28		0.26		0.10		0.17
	5		2.65		11.59		1.90		2.65		10.47		0.51		10.28
A✓R✓	4	294	2.95	105	29.45	111	2.45	292	2.67	84	16.60	61	3.02	105	15.83
	3		5.91		22.29		7.59		4.70		18.56		32.22		23.36
	2		33.95		24.41		56.94		39.32		31.35		52.03		30.11
	1		54.23		12.17		30.98		50.38		22.76		12.11		20.25
	A & B		0		0.28		0.59		0		0.14		0.16		0.37
	5		18.57		2.25		3.72		0		2.62		3.96		2.94
A×R×	4	11	2.94	166	5.32	137	5.08	9	1.20	142	5.11	172	6.66	164	5.05
	3		4.87		11.66		8.19		21.08		11.47		15.97		13.08
	2		20.59		46.73		43.13		44.68		46.84		43.92		40.28
	1		53.02		33.78		39.29		33.04		33.83		29.33		38.29
	A & B		0		0.49		0.28		0		0.60		0.10		0.09
	5		12.66		3.76		4.32		3.53		5.77		4.20		3.87
A✓R×	4	5	10.72	21	10.00	39	2.75	6	6.78	47	8.44	45	6.89	27	8.95
	3		5.19		24.97		9.42		8.93		11.20		13.80		16.70
	2		37.17		37.06		42.57		31.30		41.60		45.85		50.22
	1		34.27		23.72		40.66		49.49		32.41		29.16		20.16
	A & B		-		0		0.25		0		0.33		0.15		0
	5		-		9.73		2.74		0		5.45		0.42		7.35
A×R✓	4	0	-	14	38.98	21	4.46	1	0	33	7.17	26	3.25	10	13.16
	3		-		22.49		7.04		2.18		11.69		25.87		20.10
	2		-		20.85		51.82		34.15		34.87		50.05		39.79
	1		-		7.94		33.69		63.62		40.49		20.25		19.60

Special case for divisor 6. In the first assessment, some learners stated the divisibility rule for divisor 6 correctly, but when they were prompted for more details, it became evident that they only inspected the last one or two digits of the dividend. The dividends for divisor 6 were compiled such that inspection of only the last or last two digits would result in an incorrect answer.

Table 4 indicates the percentage of fixation time on the digits for divisor 6 where learners provided the correct divisibility rule before and after revision, but indicated an incorrect answer before revision (A×R✓) and a correct answer after revision (A✓R✓). The results of a within-subjects repeated-measures ANOVA for the effect of revision on percentage of fixation time are also shown.

Significant p values (α = .05) were found for all the digits except for digit 2. Therefore, it can be inferred that some learners who knew the divisibility rule for divisor 6, applied the rule incorrectly (did not actually focus on all digits) before revision. One can, therefore, infer that the percentage of fixation time on digits can indicate if a learner applied the rule correctly.

Table 4: The effect of revision on the percentage of fixation time per digit for divisor 6 for learners in A×R✓ before revision and A✓R✓ after revision. (N=number of responses)

Divisor	Digit	N	Percentage of fixation time Assessment 1 A×R✓	Assessment 2 A✓R✓	*p*
	A & B		0.43	0.15	.569
	5		2.33	9.81	.033
	4		3.68	18.00	.018
6	3	11	6.04	15.71	.004
	2		32.51	26.39	.371
	1		55.01	29.93	.004

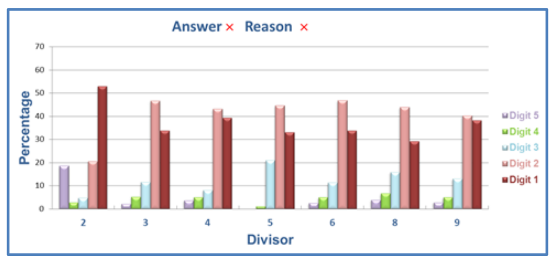

Figure 2: Percentage of total fixation time per divisor and digit for learners in School A in A×R× (Both assessments)

6.2 Percentage of Fixation Time per digit for Learners in A×R×

A trend was observed that if a learner provided an incorrect answer and did not know the divisibility rule, the highest percentage of fixation time was mainly on the last two digits (digit 1 and digit 2). Although they indicated verbally that they only inspected the last digit of the dividend, it was revealed through eye-tracking that there was also a high percentage of fixation time on the second-to-last digit.

Information gained from participants is not very objective and eye-tracking can be a useful complementary source of information to identify the strategies that participants use when solving a problem [24]. Therefore, when teachers compile dividends to be used in the assessment of divisibility rules, they have to keep in mind that learners who do not know the divisibility rules, mainly inspect the last two digits of the dividend.

The average percentage of fixation time per digit for learners who provided the incorrect answer and also the incorrect divisibility rule (A×R×) is shown in Table 3. Figure 2 provides a visualisation of the results for learners in A×R×. A series of one-way analyses of variance (ANOVA) showed a significant effect of the digit position on the percentage of fixation time for each divisor. This means that learners were deliberately focusing on the last two digits irrespective of what the divisor was.

7 MINIMUM ATTENTION REQUIRED PER DIVISOR

Table 5 shows the average percentage of fixation time that learners who provided the correct answer and reason spent per digit. The last column in the table indicates the lower limit of the 95% confidence interval, i.e. the percentage of which we can be 95% sure that learners in A✓R✓ spent at least this much time on the respective digits. Using these values and the trends that were discovered above, the minimum required attention levels per digit can be determined for each divisor. Based on these levels, a teacher will be able to tell if the respective divisibility rule was applied correctly.

7.1 Minimum Attention Required per Divisor

7.1.1 Divisors 2 and 5. A dividend is divisible by 2 if the last digit is even, and divisible by 5 if the last digit is zero (0) or five (5). Since both the rules for divisors 2 and 5 expect the learners to focus on the last digit only, it could be argued that the same criteria should be applied for these two divisors. Table 5 indicates that the average percentage of fixation time that learners spent on the last digit for divisor 2 and 5 was 54.23% and 50.38% respectively and that we can be 95% sure that the percentage of fixation time will be higher than 51.04% and 47.08% respectively.

The minimum percentage of fixation time of 47% will be acceptable for divisors 2 and 5. Therefore, one can surmise that for divisibility by divisors 2 and 5 learners had to spend 47% of their time on the last digit to be 95% sure that the divisibility rule was applied correctly.

7.1.2 Divisor 4. A dividend is divisible by 4 if the number formed by the last two digits is divisible by 4. Table 5 indicates that the average percentage of fixation time that learners in A✓R✓ spent on the last two digits for divisor 4 was 87.92% and that we can be 95% sure that the total fixation time on the last two digits will be higher than 80.42%. Therefore, one can surmise that for divisibility by 4, learners had to fixate on both digit 1 and digit 2, and they had to spend 80% of the time on the last two digits to be reasonably sure that the learner applied the divisibility rule correct.

7.1.3 *Divisor 8.* A dividend is divisible by 8 if the number formed by the last three digits is divisible by 8. Table 5 indicates that the average percentage of fixation time that learners in A✓R✓ spent on the last three digits for divisor 8 was 96.36% and that we can be 95% sure that the total fixation time on the last three digits will be higher than 83.34%. All the learners in A✓R✓ fixated on digit 2 and digit 3, but 9% of these learners did not fixate on digit 1. This agrees with

Table 5: Percentage of fixation time per divisor and digit for learners in A✓R✓. (Learners should inspect the underlined digits according to the divisibility rule)

Divisor	N	Digit	Percentage fixation time: %	Percentage fixation time: Lower limit of 95% confidence interval
2	294	A&B	0.32	0.17
		5	2.65	1.81
		4	2.95	2.13
		3	5.91	4.82
		2	33.95	31.16
		<u>1</u>	54.23	51.04
3	105	A&B	0.09	0.02
		<u>5</u>	11.59	9.31
		<u>4</u>	29.45	26.19
		<u>3</u>	22.29	19.88
		<u>2</u>	24.41	21.82
		<u>1</u>	12.17	10.19
4	111	A&B	0.15	0.02
		5	1.90	0.82
		4	2.45	1.40
		3	7.59	5.13
		<u>2</u>	56.94	53.20
		<u>1</u>	30.98	27.22
5	292	A&B	0.28	0.11
		5	2.65	1.74
		4	2.67	1.86
		3	4.70	3.46
		2	39.32	36.31
		<u>1</u>	50.38	47.08
6	84	A&B	0.26	0.03
		<u>5</u>	10.47	8.71
		<u>4</u>	16.60	14.04
		<u>3</u>	18.56	16.42
		<u>2</u>	31.35	28.27
		<u>1</u>	22.76	19.00
8	61	A&B	0.10	0.04
		5	0.51	0.14
		4	3.02	1.31
		<u>3</u>	32.22	27.37
		<u>2</u>	52.03	47.27
		<u>1</u>	12.11	8.70
9	105	A&B	0.17	0.04
		<u>5</u>	10.28	8.38
		<u>4</u>	15.83	13.96
		<u>3</u>	23.36	20.97
		<u>2</u>	30.11	27.55
		<u>1</u>	20.25	17.61

where it can be seen in Table 5 that the percentage of fixations on the last digit was much lower than on digits 2 and 3. Therefore, one can surmise that for divisibility by divisor 8, learners had to spend 80% of the time on the combined last three digits for the researcher to be 95% sure that the learner applied the divisibility rule correctly, and the learners also had to fixate on digit 2 and digit 3.

7.1.4 Divisor 3, 6 and 9. A dividend is divisible by 3 if the sum of the digits is divisible by 3 which means that a learner should inspect all the digits. The percentage of fixation time on the digits vary between the two questions that were asked, but all digits enjoyed at least 10% of the attention. The smallest percentage of fixation time was on digits 1 and 5. It is understandable that there will be a low percentage of fixation time on the first digit that learners inspect, regardless of whether they start from the left or right, because they only have to remember the first digit. Thereafter, learners have to accumulate the values of the digits – thus adding to the mental effort with consequential longer fixations. Although every digit enjoyed at least 10% attention, it will not be justified to require a minimum percentage of fixation time per digit. If a digit is zero (0) or one (1), learners will quickly fixate on it and move on. It is, however, important that learners fixate on all the digits.

A dividend is divisible by 6 if the dividend is an even number and the sum of the digits is divisible by 3, which means that the learner should inspect all the digits as for divisibility by 3. Because of the general trend that learners who do not know the divisibility rule fixate mainly on the last two digits, it is important that the dividend is even when testing divisibility by 6.

A dividend is divisible by 9 if the sum of the digits is divisible by 9. This means that learners had to inspect all digits. The same arguments that hold for divisor 3 also apply to divisor 9.

Although learners should inspect all the digits for divisors 3, 6 (if it is even) and 9, it was found that some learners in A✓R✓ did not do so. Some learners fixated on four of the five digits only. It could be argued that the fixations were too short to be registered by the eye-tracker (as in cases where easy digits, such as 0 or 1, were not fixated on), or it could be that digits are perceived in learners' peripheral vision without explicit fixations. It could, therefore, be argued that in order to determine divisibility by 3, 6 (if the dividend is even) or 9, learners have to fixate on at least four of the five digits.

7.2 Validation of the Minimum Required Attention Levels

To validate the required minimum attention levels that were set, all the eye-tracking recordings with the learners' answers and reasons were used to compare these levels with the reasons that learners provided. Although recordings were discarded in the analysis of data where there were no fixations on the areas of interest, it was used during the validation because it could happen in practice that there are no fixations on the AOIs. Table 6 shows the percentage of responses in each of the possible answer/reason/gaze combinations combined for all divisors per grade for the two assessments. Figure 3 provides a visualisation of the results in Table 6.

If a learner provides an incorrect answer, a teacher must determine the reason, because it is possible that the learner (i) does not know the divisibility rule (Figure 3a), (ii) knows the divisibility rule but applied it incorrectly (Figure 3c), or (iii) knows the divisibility rule but made calculation errors (Figure 3d). For divisors 3, 6 and 9, the percentage of fixations could readily indicate which one of these categories applies. For the other divisors, it is less obvious since the trend that learners who do not know the divisibility rule focus on the last two digits, overlaps with the correct gaze behaviour. Figure 3b illustrates this trend, because there were a percentage of responses in A×R× with acceptable gaze behaviour. It is therefore important that the teacher prompts the learners for a motivation if their answers are incorrect.

In cases where learners does not know the divisibility rule but provided the correct answer (Figure 3e and 3f) they probably guessed the correct answer. False rejects would occur when the learners' gazes (G) show too little attention on the respective digits but both the answer (A) and reason (R) is correct (example Figure 3g). False accepts can occur when the relevant digits enjoy an acceptable amount of attention but the answer and/or the motivation is incorrect. The overall success rate where learners provided the correct answer and where the attention corresponded with the minimum required attention levels (or the lack thereof in A✓R×), was 85.74%. In 8.6% of the cases, incorrect applications of the divisibility rules were predicted to be correct and in 5.7% of the cases, correct applications were predicted to be incorrect. The false accepts and false rejects for incorrect answers were not calculated, because the learners were not going to receive any marks for incorrect answers.

8 SUMMARY AND CONCLUSIONS

When teachers assess learners by using true/false ("yes" or "no") questions, they do not have access to the learners' reasoning. This study investigated whether eye-tracking can assist teachers in discovering learners' reasoning while they are thinking about their answers.

Learners were presented with a series of five-digit dividends on a computer screen while they had to determine whether the dividends were divisible by divisors 2, 3, 4, 5, 6, 8 and 9.

After revising the divisibility rules, the assessment was repeated for the same learners and the effects of learners' grade, revision, divisor and correctness of answer on gaze behaviour were determined through a series of within-subject analyses of variance.

It was found that learners' grade had no significant effect on the percentage of time that learners fixate on the respective digits. Revision proved to be a significant indicator of gaze behaviour for learners who improved their performance after revision but not for learners who knew the divisibility rules before revision. The divisor affected gaze behaviour significantly as it is not always necessary to inspect all digits. Learners who could not explain the divisibility rule, spent most attention on the last two digits – irrespective of whether the answer was correct (probably guessed) or not. With the exception of divisor 6, no difference in gaze behaviour was detected for learners who explained the rule correctly – again irrespective of whether the answer was correct or incorrect (probably due to a calculation error). One can, therefore, infer that the percentage of fixation time on digits can indicate if a learner applied the rule correctly.

For each divisor, a minimum percentage of fixation time per digit were determined that could be used by teachers to tell if the respective divisibility rule was applied correctly. It was found that for the learners who provided the correct answer in this study, these values were accurate in about 85% of the cases to indicate whether the learners also applied divisibility rule correctly.

In summary, it is concluded that when a teacher is in possession of the learner's answer, reason and fixation data, the teacher is in a position to identify if the learner (i) guessed the answer, (ii) correctly applied the divisibility rule, (iii) correctly applied the divisibility rule but made mental calculation errors, or (iv) incorrectly applied the divisibility rule. Eye-tracking can assist

Table 6: Percentage of responses in each of the possible answer/reason/gaze combinations combined for all divisors per grade for the two assessments. (Key: A=Answer; R=Reason; G=Gaze behaviour; ×=Incorrect/Inadequate; ✓=Correct/Adequate)

Grade	Percentage															
	Assessment 1								Assessment 2							
	××× ARG	××✓ ARG	×✓× ARG	×✓✓ ARG	✓×× ARG	✓×✓ ARG	✓✓× ARG	✓✓✓ ARG	××× ARG	××✓ ARG	×✓× ARG	×✓✓ ARG	✓×× ARG	✓×✓ ARG	✓✓× ARG	✓✓✓ ARG
4	41.8	20.0	0.0	0.4	5.7	5.0	1.4	25.7	26.4	14.3	0.4	3.9	5.0	7.1	2.5	40.4
5	27.0	25.4	2.8	2.8	4.4	4.8	3.6	29.4	12.7	17.9	0.8	2.8	2.0	6.3	3.2	54.4
6	30.7	23.9	2.5	2.5	2.5	5.0	3.6	29.3	8.9	12.9	0.4	6.8	2.9	0.7	3.2	64.3
7	18.6	17.9	0.7	2.9	5.7	6.4	4.3	43.6	3.9	5.4	1.1	5.4	2.1	3.9	4.3	73.9

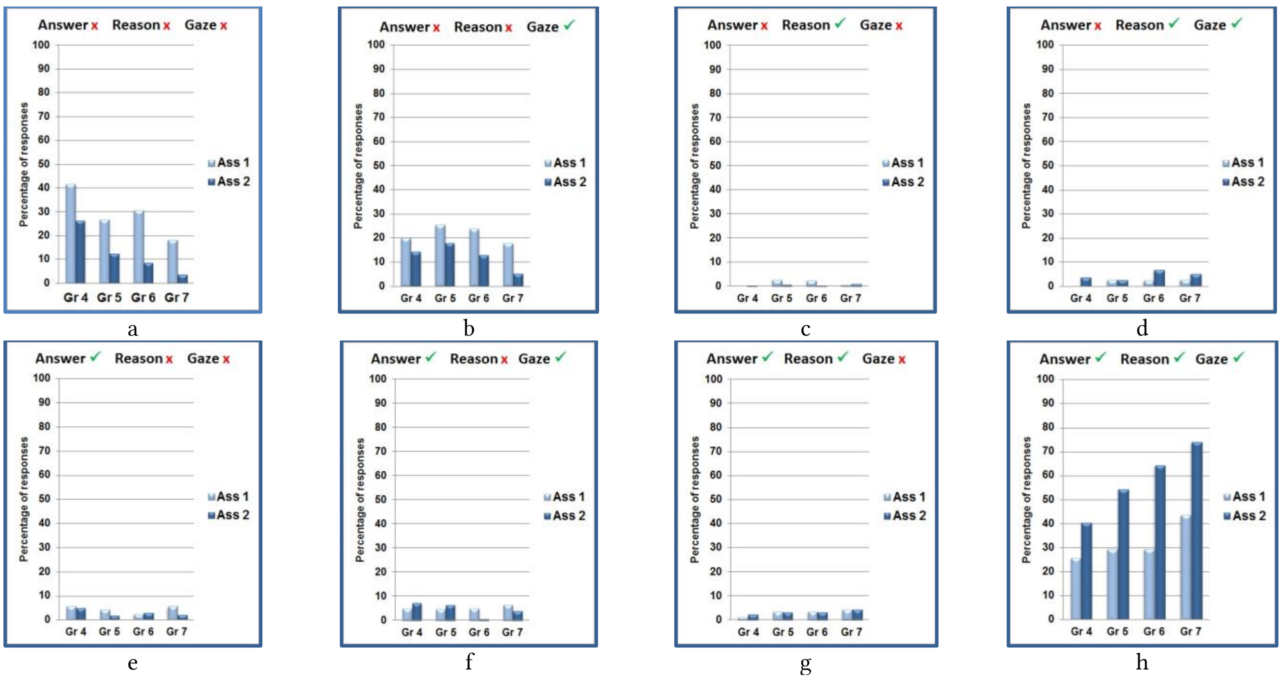

Figure 3: Percentage of responses per grade, answer, reason and gaze for all the divisors with minimum gaze requirements

teachers in identifying whether learners applied the divisibility rules correctly when their answers were correct. However, due to the relatively high probability of error, eye-tracking cannot be used exclusively to determine if learners applied the divisibility rules correctly or not.

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