

# COMPARISON OF THE PERFORMANCE OF SENSITIVITY-BASED VOLTAGE CONTROL ALGORITHMS IN DG-INTEGRATED DISTRIBUTION SYSTEMS.

P.T. Manditereza \* and R.C. Bansal \*\*

\* Dept. of Electrical, Electronic and Computer Engineering, Central University of Technology, Free State, South Africa

\*\* Dept. of Electrical, Electronic and Computer Engineering, University of Pretoria, South Africa

**Abstract:** The integration of renewable energy generators in distribution grids has increased the complexity of the voltage control problem. Reactive power control (RPC) algorithms based on sensitivity analysis have been proposed in the literature for the management of the voltage problem. Sensitivity methods are computationally complex for practical real-time analysis and this has led to use of de-coupled and other simplified load flow models. However, algorithms based on decoupled models have been shown to be inefficient for analysis of distribution systems with low X/R ratio. This paper uses a simplified line modelling approach recently proposed in the literature to facilitate the development of computationally simple distributed, non-decoupled, load flow equations that completely capture the characteristics of the radial distribution feeder, removing the need to use the decoupled models. Results show that the simple algorithm based on this new line modelling approach gives better voltage control performance compared to the decoupled models.

**Key words:** Distribution line model, distributed control, voltage control, multi-agent systems, renewable distributed generation.

## 1. INTRODUCTION

The energy policies that are being promulgated by governments world-wide to promote the exploitation and use of renewable energy resources [1], and the parallel de-regulation of the energy sector that now permits open and non-discriminatory access for small and medium independent power producers (IPPs) to the national grids [2], has seen an increasing number of distributed generation (DG) units based on renewable energy being integrated into the power grids. This increasing integration of DGs has raised a number of technical concerns including voltage regulation, raising the risk of violation of acceptable voltage limits [3-6].

The voltage control problem has generated interest in reactive power control (RPC) as a current research topic on DG-integrated distribution systems [7-10] and many algorithms based on sensitivity analysis have been proposed in the literature. Sensitivity analysis transforms the complex and nonlinear relationship between network power and voltages to a linearised model that can be used to compute the expected small changes in voltage magnitude and angle ( $V$  and  $\delta$ ) for small changes in active and reactive power ( $P$  and  $Q$ ), about a certain operating point. The sensitivity method thus provides a straightforward determination of the  $P$  and  $Q$  injections required to correct a given voltage deviation [12]. The method also provides direct information on the control node with the most significant impact on the voltage and the corresponding  $P$  and/or  $Q$  to be injected. The sensitivity method is also particularly suitable for distributed control algorithms as the localised

computational load is lower than for the centralised case [12].

The classical Jacobian matrix, derived from the Newton-Raphson load flow algorithm, has been widely used as source of the sensitivity data. However, the sensitivity method based on the Jacobian matrix is computationally complex for practical real-time power flow analysis [13]. The complexity of this process has led to the use of decoupled load flow model that assumes weak coupling between  $P$ - $V$  and  $Q$ - $\delta$ , simplifying the Jacobian matrix as only investigations of the  $P$ - $\delta$  and  $Q$ - $V$  sensitivities are necessary.

The weak coupling between  $P$ - $V$  and  $Q$ - $\delta$  has been shown in [14] to apply to the transmission system that has a high X/R ratio, for which injection of reactive power affects mostly the voltage magnitude and injection of active power affects mostly the phase angle [7]. This simplification, however, may not apply for the distribution system which has a low X/R ratio, and for which injection of active power also has a significant impact on voltage magnitude [7]. The non-decoupled model, therefore, needs to be used for developing the sensitivity matrix [7, 14].

Nonetheless, various works such as [15-17] have been presented in the literature that developed classical sensitivity based voltage control algorithms for the distribution system that take advantage of the computationally simpler decoupled model characteristics. Authors in [16] developed a subgradient based voltage control algorithm that makes further simplifying assumptions of same-bus sensitivity; that is, changes in  $P$

and Q at a node result in changes in  $\delta$  and V, respectively, at that node only. However, ignoring the interactions between the multiple nodes may lead to convergence issues. Same-bus sensitivity analysis is also used in [18] as an integral part of a control strategy to minimise voltage fluctuations. Researchers [13, 19] developed non-classical, low complexity, sensitivity-based algorithms that consider the impact of changes to P and Q on voltage magnitude V only; the changes in phase shifts are neglected. The justification being that the problem is control of the voltage magnitude and not the phase shift [7]. However, the impact of  $\delta$  on the interdependency of P-V and Q- $\delta$  are lost. Work presented in [20] develops a centralised voltage control method based on optimal generation dispatch and employing the classical Jacobian but with only the P-V and Q-V sensitivities considered for estimation of the voltage magnitude.

Simplified control algorithms can be derived by considering the structure of the distribution system itself. The radial distribution system is structurally different from a transmission system and may be modelled in a simplified way. This leads to load flow equations specifically for the radial distribution system that are much more efficient than general purpose methods. The load flow formulation developed in [21] captures the topological structure of the distribution system in matrices that describe the relationships between bus current injections at the system buses, branch currents and system voltages. The work presented in [7] develops a sensitivity-based voltage control method based on the load flow formulation in [21]. However, this method is suitable for centralised approach as it requires knowledge of the topology of the entire network.

The work presented in this paper builds on the previous work of the authors presented in [22]. This previous work takes the advantage of the simple structure of the radial distribution system to develop a line model suitable for implementation of distributed control algorithms. The model splits the distribution feeder into a series of overlapping segments, each segment composed of only three nodes. This simplified model translates into computationally simple distributed, non-decoupled, load flow equations that correctly capture the characteristics of the radial distribution feeder and are suited to real-time distributed control.

The paper is organised as follows: Section 2 gives the simplified load flow equations and describes various voltage control algorithms based on varying simplifying assumptions to the load flow problem. Section 3 briefly describes the var dispatch approach for the voltage control problem. The performance of the various algorithms are demonstrated and compared through simulations in Section 4. The conclusions are presented in Section 5.

## 2. DISTRIBUTED VOLTAGE CONTROL

Distributed voltage control algorithms based on four variations of equations (1-2) are developed. These variations are based on the various assumptions made when simplifications to the general load flow equations are made. The performances of the four algorithms are compared to assess the validity of such simplifications when applied to the distribution system.

### 2.1 Simplified load flow equations

To obtain the power flow at a node  $k$  in a radial distribution line, it is sufficient to consider only the voltages at the two adjacent nodes [22]. This realisation greatly simplifies the analysis of the voltage problem as only three nodes are relevant for the calculation of power flow at any node of the radial system. As presented in [22], the radial distribution line can therefore be assumed to be made up of overlapping sections, with each section consisting of three nodes. The overlapping structure facilitates distributed control using the multi-agent concept [23, 24] through information exchange between agents located in the line segments.

The change in active and reactive power at node  $k$ , for the 3-node segment, may be expressed as [22]:

$$\Delta P_k = \sum_j \frac{\partial P_k}{\partial \delta_j} \Delta \delta_j + \sum_j \frac{\partial P_k}{\partial V_j} \Delta V_j, \text{ for } j = (k-1, k, k+1) \quad (1)$$

$$\Delta Q_k = \sum_j \frac{\partial Q_k}{\partial \delta_j} \Delta \delta_j + \frac{\partial Q_k}{\partial V_j} \Delta V_j, \text{ for } j = (k-1, k, k+1) \quad (2)$$

Where:

$\Delta P_k$  = change in active power at node  $k$

$\Delta Q_k$  = change in reactive power at node  $k$

$\Delta \delta_j$  = change in phase angle at each node of the 3-node segment

$\Delta V_j$  = change in voltage magnitude at each node of the 3-node segment

Equations (1) and (2) are the non-decoupled equations describing the relationships between P, Q, V and  $\delta$  for the 3-node line segment. Various simplifying assumptions as introduced in Section 1 are made on equations (1-2) to give four different voltage control algorithms described in Cases 1-4 in the following:

### 2.2 Case 1: Using the non-decoupled model

From the non-decoupled equations (1) and (2), it can be shown that [21]:

$$\Delta V_k = \left[ \left( \frac{\partial P_k}{\partial \delta_k} \right)^{-1} \left( \frac{\partial P_k}{\partial V_k} \right) - \left( \frac{\partial Q_k}{\partial \delta_k} \right)^{-1} \left( \frac{\partial Q_k}{\partial V_k} \right) \right]^{-1} \left( \left[ \left( \frac{\partial P_k}{\partial \delta_k} \right)^{-1} \left[ \Delta P_k - \sum_j \frac{\partial P_k}{\partial \delta_j} \Delta \delta_j - \right. \right. \right.$$

$$\left. \left. \left. \left. \left. \sum_j \frac{\partial P_k}{\partial V_j} \Delta V_j \right] \right] - \left[ \left( \frac{\partial Q_k}{\partial \delta_k} \right)^{-1} \left[ \Delta Q_k - \sum_j \frac{\partial Q_k}{\partial \delta_j} \Delta \delta_j - \sum_j \frac{\partial Q_k}{\partial V_j} \Delta V_j \right] \right] \right] \right), \text{ for } j = (k-1, k+1) \quad (3)$$

It can be seen from (3) that  $\Delta V_k$  is zero if the following condition is satisfied:

$$\left( \frac{\partial P_k}{\partial \delta_k} \right)^{-1} \left[ \Delta P_k - \sum_j \frac{\partial P_k}{\partial \delta_j} \Delta \delta_j - \sum_j \frac{\partial P_k}{\partial V_j} \Delta V_j \right] = \left( \frac{\partial Q_k}{\partial \delta_k} \right)^{-1} \left[ \Delta Q_k - \sum_j \frac{\partial Q_k}{\partial \delta_j} \Delta \delta_j - \sum_j \frac{\partial Q_k}{\partial V_j} \Delta V_j \right] \quad (4)$$

With the active power held constant, only the reactive power can be adjusted to mitigate voltage effects of connection of DG. From (4), the reactive power compensation required is,

$$\Delta Q_k = - \left( \frac{\partial Q_k}{\partial \delta_k} \right) \left( \frac{\partial P_k}{\partial \delta_k} \right)^{-1} \left[ \sum_j \frac{\partial P_k}{\partial \delta_j} \Delta \delta_j + \sum_j \frac{\partial P_k}{\partial V_j} \Delta V_j \right] + \left[ \sum_j \frac{\partial Q_k}{\partial \delta_j} \Delta \delta_j + \sum_j \frac{\partial Q_k}{\partial V_j} \Delta V_j \right], \text{ for } j = (k-1, k+1) \quad (5)$$

The formulae for calculating the sensitivity coefficients can be found in [22].

### 2.3 Case 2: Using the decoupled model

Decoupling implies weak P-V and Q- $\delta$  sensitivities. In this case (1) and (2) are modified to:

$$\Delta P_k = \sum_j \frac{\partial P_k}{\partial \delta_j} \Delta \delta_j, \text{ for } j = (k-1, k, k+1) \quad (6)$$

$$\Delta Q_k = \sum_j \frac{\partial Q_k}{\partial V_j} \Delta V_j, \text{ for } j = (k-1, k, k+1) \quad (7)$$

That is, injection of active power affects mostly the phase angle and injection of reactive power affects mostly the voltage magnitude.

From (7),

$$\Delta V_k = \left( \frac{\partial Q_k}{\partial V_k} \right)^{-1} \left( \Delta Q_k - \sum_j \frac{\partial Q_k}{\partial V_j} \Delta V_j \right), \text{ for } j = (k-1, k+1) \quad (8)$$

It can be seen from (8) that  $\Delta V_k$  is zero if the following condition is satisfied:

$$\Delta Q_k = \sum_j \frac{\partial Q_k}{\partial V_j} \Delta V_j, \text{ for } j = (k-1, k+1) \quad (9)$$

Equation (9) gives the reactive power compensation required to eliminate  $\Delta V_k$  for the decoupled case.

### 2.4 Case 3: Assuming same-bus sensitivity

A further simplifying assumption to decoupling, same-bus sensitivity, is that changes in P and Q at a node result in changes in  $\delta$  and V, respectively, at that node only. In this case (1) and (2) are reduced to:

$$\Delta P_k = \frac{\partial P_k}{\partial \delta_k} \Delta \delta_k \quad (10)$$

$$\Delta Q_k = \frac{\partial Q_k}{\partial V_k} \Delta V_k \quad (11)$$

Equation (11) directly gives the reactive power compensation required to eliminate  $\Delta V_k$  for this case.

### 2.5 Case 4: Considering only the V-P-Q relationship

Other simplifying assumptions consider the impact of changes to P and Q on voltage magnitude V only; the changes in phase shifts are not considered. That is,

$$|\Delta V| = \frac{\partial V}{\partial P} \Delta P + \frac{\partial V}{\partial Q} \Delta Q \quad (12)$$

Hence, for the 3-node line segment and with power injection at node k only, (12) can be re-written as:

$$|\Delta V|_k = \frac{\partial V_k}{\partial P_k} \Delta P_k + \frac{\partial V_k}{\partial Q_k} \Delta Q_k \quad (13)$$

The impact of  $\delta$  on V is lost.

From (13), and with the active power held constant, the total reactive power compensation required to eliminate  $\Delta V_k$  for this case is given by,

$$\Delta Q_k = \sum \left( \frac{\partial V_k}{\partial Q_k} \right)^{-1} \Delta V_k \quad (14)$$

## 3. REACTIVE POWER DISPATCH

The reactive power compensations for the four cases are given by equations (5), (9), (11) and (14), respectively. The calculations of the reactive compensations are performed by the agents distributed in each of the line segments, using the voltage magnitudes and angles received from the two neighbouring agents. Compensation at one node will affect the voltages at all the other nodes. Therefore, except for Case 3, the calculated compensations are not mutually exclusive, i.e. they cannot be applied simultaneously. Hence only one of these should be applied at a time. The dispatch of the calculated MVar compensation is facilitated by identifying the DG with the maximum impact on the voltage profile [13, 22] to ensure that minimum vars are deployed to eliminate the voltage errors. The reactive power compensation is adjusted, iteratively, until the voltage deviation from base case is eliminated.

For case 3, it is assumed that compensation at a node result in voltage change at that node only. Hence, the reactive compensations at all controllable nodes can be applied simultaneously.

## 4. SIMULATION RESULTS AND DISCUSSION

In the distributed algorithms described in this paper, each agent needs to process only a small amount of data for each iteration of control update. The authors in [16]

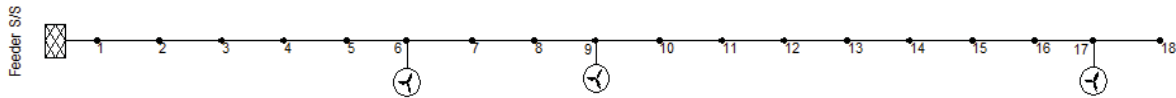


Figure 1: The 18-node network considered in the simulation

identified an update cycle of 2 seconds as reasonable and more than sufficient to complete the calculations for the particular distributed control algorithm described in their work. This cycle time of 2 seconds is adopted and applied for the simulations in this paper.

The performance of the various algorithms are demonstrated and compared through DigSilent *Powerfactory* software simulations on a 22kV, 18-node radial distribution network shown in Fig. 1. The system data is given in the Appendix.

Two sets of tests are performed on this network, one with a low X/R ratio of 0.55 and the other with a high X/R ratio of 5. The ratios are suggested according to typical line parameters given in [25]. Three DGs are randomly distributed at nodes 6, 9 and 17.

The first test gives a base voltage profile shown in Fig. 2. The voltage varies from 1.02 p.u. at the feeder substation to 0.98 p.u. at the last node, 18. Fig. 2 also shows the voltage profile after a 2.3 MW DG is connected at node 17 through a step-up transformer. The voltage profile increases substantially reaching 1.07 p.u. at node 17, beyond the acceptable upper limit of 1.05 p.u. As a result, voltage control must be deployed to resolve the overvoltage problem.

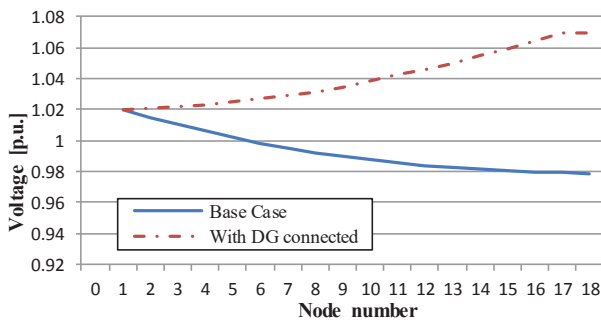


Figure 2: Comparison of voltage profile for base case and with DG connected.

Voltage control algorithms based on the three system models are tested and their performances are compared: Case 1 – voltage control with the non-decoupled model; Case 2 - voltage control with the decoupled model; Case 3 - voltage control with the same-bus sensitivity model. With only one controllable node (DG activated at node 17 only), Case 4 is equivalent to Case 3 when active power curtailment is not applied. Reactive power compensation is activated at node 17 for each case.

Fig. 3 shows the voltage response at node 17 to the three algorithms (Cases 1-3) with a test network of low X/R

ratio of 0.55. It can be seen that the voltage converges faster when the non-decoupled model is used. Use of the decoupled and same-bus sensitivity models show a sluggish voltage response with the voltage converging to the base profile value after 25 seconds, compared to the convergence time of about 10 seconds for the non-decoupled case.

The voltage response at all the nodes are described in Fig. 4 (a-c), giving further illustration of the differences between the performances of the non-decoupled model and the other two cases.

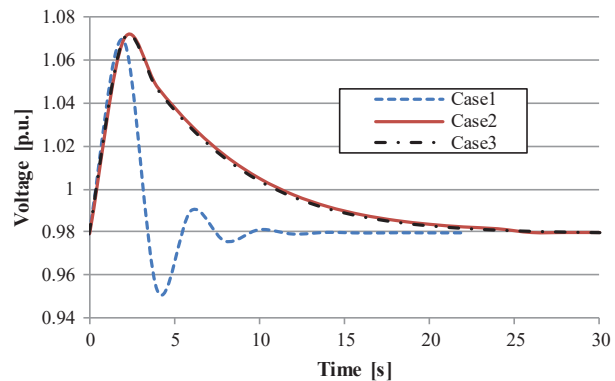
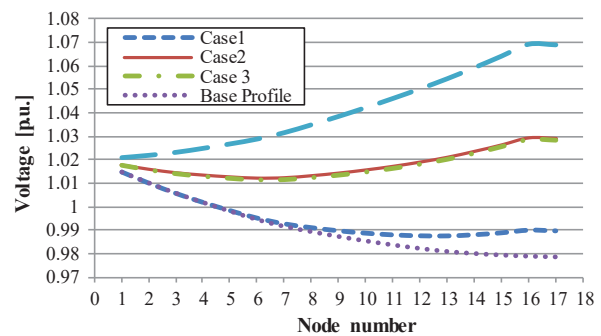
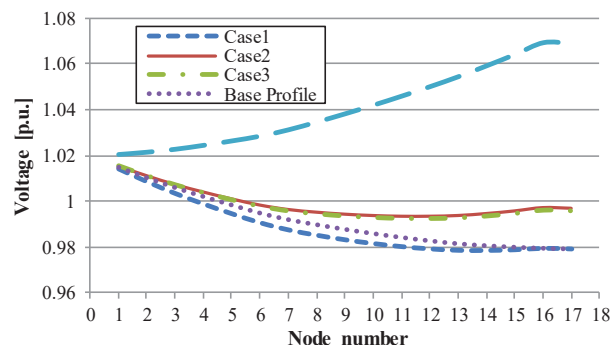


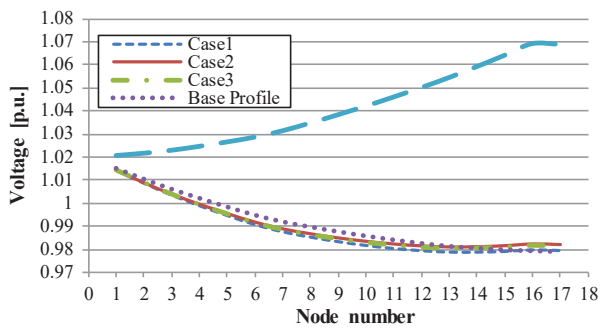
Figure 3: Comparison of voltage response under the non-decoupled and decoupled system model descriptions



(a) Comparison of voltage profiles after 4 seconds



(b) Comparison of voltage profiles after 10 seconds



(c) Comparison of voltage profiles after 20 seconds

Figure 4(a-c): Comparison of the evolution of the voltage profiles at different time instants.

As can be seen in Figures 4 (a-b), the voltage profile approaches the base case profile faster, in about 10 seconds, when the non-decoupled model is used. The voltage profile with the decoupled model only approaches the base case after 20 seconds. This further emphasizes the sluggish response when the decoupled model is applied to the distribution system with low X/R ratio.

In order to verify that the slower response of the decoupled and same-bus models is due to the low X/R ratio, the voltage control algorithms are again tested on the system with X/R ratio increased to 5. The voltage response at node 17 is shown in Fig. 5. It can be seen that the performance of the decoupled model (Case 2) is now similar to that of the non-decoupled model (Case 1). (The plot for Case 1 is directly aligned with that for Case 3 – same bus sensitivity model - in Fig. 5). This proves that, in this instance, no advantage is obtained by using the computationally complex non-decoupled model. The decoupled model is thus used for systems with high X/R ratio with no loss of performance or accuracy. But it has been shown that the decoupled model is not efficient for the low X/R ratio system. The line model introduced in [22] simplifies the implementation of the non-decoupled model that is seen to give better performance for distribution systems of low X/R ratio.

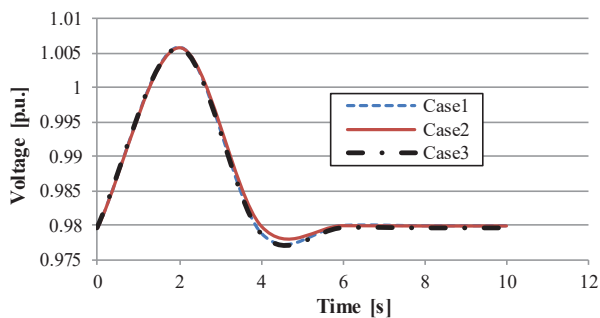


Figure 5: Comparison of voltage response under the non-decoupled and decoupled system model descriptions for high X/R ratio.

## 5. CONCLUSION

The use of decoupled and other simplified load flow models have been shown to be unsuitable for the distribution system with low X/R ratio. Use of the simple distributed line model facilitates the development of computationally simple distributed, non-decoupled, load flow equations that completely capture the characteristics of the radial distribution feeder. A voltage control algorithm based on this model gives better voltage control performance compared to use of the decoupled models.

## 6. APPENDIX

Table 1: The line and load data for the 18-node network.

Id	Node A	Node B	R (Ohms)	X (Ohms)	Load (Node B)	
					kW	kVAr
1	1	2	1.25	0.6875	100	60
2	2	3	1.25	0.6875	90	40
3	3	4	1.25	0.6875	120	80
4	4	5	1.25	0.6875	60	30
5	5	6	1.25	0.6875	200	100
6	6	7	1.25	0.6875	200	100
7	7	8	1.25	0.6875	60	20
8	8	9	1.25	0.6875	60	20
9	9	10	1.25	0.6875	45	30
10	10	11	1.25	0.6875	60	35
11	11	12	1.25	0.6875	60	35
12	12	13	1.25	0.6875	120	80
13	13	14	1.25	0.6875	60	20
14	14	15	1.25	0.6875	120	80
15	15	16	1.25	0.6875	60	30
16	16	17	1.25	0.6875	60	20
17	17	18	1.25	0.6875	90	40

Table 2: Model parameters of the 2.3 MW asynchronous generator.

Parameter	Value	Unit
Rated voltage	690	V
Magnetising inductance, $L_m$	$2.5 \times 10^{-3}$	H
Rotor leakage inductance, $L_{rr}$	$87 \times 10^{-6}$	H
Stator leakage inductance, $L_{ss}$	$87 \times 10^{-6}$	H
Rotor resistance, $R_r$	0.026	$\Omega$
Stator resistance, $R_s$	0.029	$\Omega$
Inertia constant, H	1.5	s

## 7. REFERENCES

- [1] C.C. Fung, S.C. Tang, Z. Xu and K.P. Wong, "Comparing Renewable Energy Policies in Four Countries and Overcoming Consumers' Adoption Barriers with REIS," in IEEE Power and Energy Society General Meeting (PES), pp. 1-5, Jul 2013.
- [2] T. Jamasab, "Between the state and the market: Electricity sector reform in developing countries," The Utilities Policy Journal, Elsevier, vol. 14, no. 1, pp. 14-30, March 2006.
- [3] P.T. Manditereza and R.C. Bansal, "Renewable distributed generation: The hidden challenges - A review from the protection perspective," Renewable and Sustainable Energy Reviews, vol. 58, pp. 1457-

1465, 2016.

- [4] N.K. Roy and H.R. Pota, "Current status and issues of concern for the integration of distributed generation into electricity networks," *IEEE Systems Journal*, vol. 9, no. 3, pp. 933-944, February 2014.
- [5] M.N. Kabir, Y. Mishra, G. Ledwich, Z. Xu and R.C. Bansal, "Improving voltage profile of residential distribution systems using rooftop PVs and Battery Energy Storage systems," *Applied Energy*, vol. 134, pp. 290-300, August 2014.
- [6] R.C. Bansal, T.S. Bhatti, and D.P. Kothari, "Automatic Reactive Power Control of Wind-Diesel-Micro-Hydro Autonomous Hybrid Power Systems Using ANN Tuned Static Var Compensator," in *Proc. Int. Conf. on Large Engineering System Conference on Power Engineering (LESCOPE)*, Montreal, Canada, pp. 182-188, 2003.
- [7] B. Bakhshideh Zad, H. Hasanvand, J. Lobry and F. Vallée, "Optimal reactive power control of DGs for voltage regulation of MV distribution systems using sensitivity analysis method and PSO algorithm," *Electrical Power and Energy Systems*, vol. 68, pp. 52-60, 2015.
- [8] A. Zakariazadeh, O. Homaei, S. Jadid and Pi Siano, "A new approach for real time voltage control using demand response in an automated distribution system," *Applied Energy*, vol. 117, pp. 157-166, 2014.
- [9] B. Bhattacharyya, S.K. Goswami and R.C. Bansal, "Loss Sensitivity Approach in Evolutionary Algorithms for Reactive Power Planning," *Electric Power Components and Systems*, vol. 37, no. 3, pp. 287-299, February 2009.
- [10] T.S. Bhatti, R.C. Bansal and D.P. Kothari, "Reactive Power Control of Isolated Hybrid Power Systems," in *Proc. of Int. Conf. on Computer Application in Electrical Engineering Recent Advances (CERA)*, Indian Institute of Technology Roorkee (India), pp. 626-632, 2002.
- [11] R.C. Bansal, T.S. Bhatti and V. Kumar, "Reactive Power Control of Autonomous Wind-Diesel Hybrid Power Systems Using ANN," in *Proc. 8th Int. Power Engineering Conf.*, Singapore, pp. 1376-1381, 2007.
- [12] F. Tamp and P. Ciufo, "A Sensitivity Analysis Toolkit for the Simplification of MV Distribution Network Voltage Management," *IEEE Transactions on Smart Grid*, vol. 5, no. 2, pp. 559-568, March 2014.
- [13] M. Brenna, E. De Berardinis, L. Delli Carpini, F. Foiadelli, P. Paulon, P. Petroni, G. Sapienza, G. Scrosati and D. Zaninelli, "Automatic Distributed Voltage Control Algorithm in Smart Grids Applications," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 877-885, June 2013.
- [14] Y. He, M. Petit and P. Dessante, "Optimization of the Steady Voltage Profile in Distribution Systems by Coordinating the Controls of Distributed Generations," in *3rd IEEE PES Innovative Smart Grid Technologies Europe (ISGT Europe)*, Berlin, pp. 1-7, 2012.
- [15] M.E. Baran and I.M. El-Markabi, "A Multiagent-Based Dispatching Scheme for Distributed Generators for Voltage Support on Distribution Feeders," *IEEE Transactions on Power Systems*, vol. 22, no. 1, pp. 52-59, February 2007.
- [16] W. Zhang, W. Liu, X. Wang, L. Liu and F. Ferrese, "Distributed Multiple Agent System Based Online Optimal Reactive Power Control for Smart Grids," *IEEE Transactions on Smart Grid*, vol. 5, no. 5, pp. 2421-2431, September 2014.
- [17] A. Cagnano and E. De Tuglie, "Centralized voltage control for distribution networks with embedded PV systems," *Renewable Energy*, vol. 76, pp. 173-185, 2015.
- [18] R. Aghatehrani and A. Golnas, "Reactive Power Control of Photovoltaic Systems Based on the Voltage Sensitivity Analysis," in *IEEE Power and Energy Society General Meeting*, San Diego, CA, pp. 1-5, 2012.
- [19] K.H. Youssef, "A new method for online sensitivity-based distributed voltage control and short circuit analysis of unbalanced distribution feeders," *IEEE Transactions on Smart Grid*, vol. 6, no. 3, pp. 1253-1260, May 2015.
- [20] C. Ma, P. Kaufmann, J.C. Tobermann and M. Braun, "Optimal generation dispatch of distributed generators considering fair contribution to grid voltage control," *Renewable Energy*, vol. 87, pp. 946-953, 2016.
- [21] J.H. Teng, "A Direct Approach for Distribution System Load Flow Solutions," *IEEE Transactions on Power Delivery*, vol. 18, no. 3, pp. 882-887, July 2003.
- [22] P.T. Manditereza and R.C. Bansal, "Multi-agent based distributed voltage control algorithm for smart grid applications," *Electric Power Components and Systems*, vol. 44, no. 20, pp. 2352-2363, December 2016.
- [23] S. Gehao, J. Xiuceng and Z. Yi, "Optimal coordination for multi-agent based secondary voltage control in power system," in *IEEE/PES Transmission and Distribution Conference*, Dalian, pp. 1-6, 2005.
- [24] N. Yorino, Y. Zoka, M. Watanabe and T. Kurushima, "An Optimal Autonomous Decentralized Control Method for Voltage Control Devices by Using a Multi-Agent System," *IEEE Transactions on Power Systems*, vol. 30, no. 5, pp. 2225-2233, September 2015.
- [25] B.M. Weedy, B.J. Cory, N. Jenkins, J.B. Ekanayake and G. Strbac, *Electric Power Systems*, 5th ed.: John Wiley, 2012.