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ORIGINAL ARTICLE

Regular classroom teachers' recognition and support of the creative potential of mildly gifted mathematics learners

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Abstract Post independent reforms in South Africa moved from separate education for the gifted learners to inclusive education in regular classrooms. A specific concern that has been totally ignored since then is whether or not the regular classroom would expand or limit the gifted child's creativity. This study aimed at investigating the extent to which South African mathematics teachers recognised and supported the development of gifted students' creative potential. Four teachers were each observed teaching over a week and the analysis focused on their representational fluency and how they responded to gifted students' creative ideas. The results show that in 70% of the episodes teachers' representations were either mathematically faulty or correct but with no further justification or explanation. In 63% of the micromoments students' creative ideas were considered disruptive and were therefore not recognized. These results suggest that currently regular classrooms in South Africa might not be conducive to the development of the gifted students' creative potential.

1 Background to the problem

Post-independent education in South Africa, as in many other developing countries, moved from separated and specialised provision for the gifted students to inclusive education where all learners were to be educated in regular classrooms (Kokot 2011). The basic assumption was that all students, regardless of their ability or disability, would benefit from schools adopting such inclusive practices. However a specific concern that has totally been ignored since 1994 is whether or not the regular classroom was indeed meeting the needs of the gifted students. While giftedness in general should be valued, the need for talent development of mathematically gifted students is even more pressing in the 21st century economy. The intuitive thought has been that mathematically gifted individuals have the potential to become the critical human capital needed for driving modern day economies. While this assumption has only been intuitive, Terman's Genetic Studies (Friedman and Martin 2011) and the longitudinal Studies of Mathematically Precocious Youth—SMPY (Lubinski et al. 2014) are arguably among the most famous longitudinal studies in psychology to date that have tracked mathematically gifted youth over decades with the aim of confirming this intuitive thought. Results from these studies have confirmed beyond any reasonable doubt that mathematically talented males and females indeed became the critical human capital needed for driving modern day, conceptual economies.

Although stakeholders have been hostile to and resentful of gifted education programs; more recently the South African National Planning Commission (NPC 2011) recommended that opportunities for excellence be provided for the most talented students. A question logically following from this recommendation would be: 'What attributes of gifted students need to be developed?' Gifted individuals are often seen as 'the hope of the future' because of the special creative attributes that they possess. Creativity and innovation are becoming increasingly important for the development of the 21st century knowledge society because they contribute to economic prosperity as well as to social and individual well being. Creativity is seen as the source of innovation, and innovation in turn as the implementation

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of creativity. Leikin (2011) therefore proposed that mathematics education must pay more attention to research of different kinds of mathematical activities, with a clear focus on students' creative thinking and giftedness. Consistent with these views, this paper aims at investigating the extent to which South African mathematics teachers' approaches supported/inhibited students' growth of such potential and whether or not teachers recognised students' creative potential. A question that follows logically from this aim is whether or not there are pedagogical practices that are appropriate for promoting gifted children's creativity. While there might be more than one effective teaching approach, there is wide consensus among researchers that the use of multiple representations (representational fluency) and the fostering of an environment that facilitates and values various representations could provide a space where learners can develop the tools to become citizens who are productive/creative and active (Brijlall et al. 2012; Krutetski 1976; Leikin 2009; Star and Newton 2009). The pedagogical implications are that teachers who are committed to supporting creative potential need to (a) make flexible use of representations as well as (b) create an environment that allows learners the freedom to use different representations. Given this duality, in this paper I address two complementary questions:

1. To what extent does teacher representational fluency support/hinder gifted learners' creativity in the regular classroom?
2. In what way do gifted students demonstrate their mathematical creativity and how do teachers respond to such creativity?

These two research questions are consistent with the long standing work of Wood, Bruner and Ross (1976) on scaffolding, within which they contend that the learner cannot benefit from such scaffolding assistance unless one paramount condition be fulfilled namely, comprehension of the solution must precede production. This paramount condition suggests that the teacher must at an early stage of the tutorial process describe or illustrate a concept, problem or process in multiple ways to ensure students' understanding before the students can be able to produce similar or more creative versions. Tharp and Gallimore (1998) refer to this as 'cognitive structuring' in which the teacher provides 'explanatory and belief structures that organise and justify'. So it is this cognitive structuring of mathematical concepts by the teachers that the first question addresses. The second research question for this paper is aimed at evaluating teachers' contingent teaching. One of the essential factors in scaffolding is 'contingent responsivity' which is described as the ability to read the child's cues and signals related to learning, affective, and motivational needs, and then to respond in a timely and appropriate way (Lidz 1991). Support that is adapted or contingent upon a student's understanding is considered effective in promoting student learning.

1.1 Theoretical framework

There is a plethora of definitions of giftedness from which no single definition or perspective has been agreed upon in the field. Given this lack of consensus the author needed a theory that would guide the study in navigating through this maze. Theory frames what and how one chooses to look at something, and according to Renzulli (2012) if we are not guided by a unified theory when choosing options, we are likely to fall for anything. Theory is therefore the rudder and compass that should guide us toward practices that avoid randomness in the goals we pursue. While there are a number of influential theories and models in the field of gifted education, Gagné's (1999) model is among the top six that have been considered dominant in affecting international classroom practice. The model has received worldwide recognition because it is generally viewed as resolving the controversies that the gifted field has struggled with for years (Pfeiffer 2013). In 1985 Gagné first conceptualised his theory of talent development which he first named as the Differentiated Model of Giftedness and Talent (DMGT). Over three decades since its inception Gagné made further refinements to the model resulting in what he now calls the Comprehensive Model of Talent Development (CMTD) (Gagné 2015). Essentially, Gagné has been dissatisfied with the frequent, all-encompassing and interchangeable use of the terms gifted and talented. He argued that the 'one term fits all' use of gifts and talents was inaccurate, misleading, and detrimental to all efforts to identify and nurture talent, because it suggests that talents are inborn hence there is no place for systematic training, learning or practicing. Yet there is ample evidence from elite sport and performing arts programs that have combined identification of ability with honing of this potential into talents. Gagné therefore argued that there is, and should be, a clear distinction between these two most basic concepts—'gifts' and 'talents'. In his CMTD model Gagné (2015) uses the term 'giftedness' to refer to the outstanding natural abilities or aptitudes—the emerging form or potential; while the term 'talented' is used to refer to the outstanding mastery of systematically developed competencies or performance. An underlying principle of Gagné's view is that while high ability (talent) has some genetic basis (giftedness), learning, practice, and environmental factors are necessary for the emergence and development of such talent. The labels 'natural' vs. 'systematically developed' used in the model point to Gagné's choice as the overarching differentiator, namely the strength of genetic input in the case of aptitudes as opposed to the capital role of practice in the case of competencies/talents. An important implication for the field of gifted education is that although the path to outstanding performance may begin with demonstrated potential, the talent associated with giftedness must be developed and sustained by way of training and interventions in domain-specific skills (Lubinski 2010).

The CMTD model then depicts the progressive development of gifts into talents in a potential-performance continuum where on one end 'gifts/natural abilities' represent the raw material and on the other end 'talents/competencies' represent the outcome of the talent development process. This developmental process is continually modulated by two large sets of catalysts which are critical in activating the translation of giftedness into talent. Central to this translation are the very important mediating effects of systematic training and practice through a structured program of activities. The concept of talent development is formally defined as the systematic pursuit by talentees, over a significant and continuous period of time, of a structured program of activities leading to a specific excellence goal (Gagné 2010). So according to Gagné (2015), if this formal developmental process is poor gifted students may never develop to their full potential. Society therefore needs to provide effective support for the gifted because their gifts may otherwise never be translated into talents. What makes Gagné's model particularly relevant for this paper on student support is the place given to learning within the developmental process. Learning implies a role for the teacher in devising programmes for pupils to follow in order to develop and improve their skills. In this regard Leikin (2011) posits that teachers are the agents of the educational system who have to design mathematical challenges appropriate for all the students in general and for the mathematically gifted ones in particular. Hence in this paper I am asking questions about how teachers support/inhibit such students' progress towards reaching their optimum level in the regular classroom.

Besides this distinction between giftedness and talent another important question that has caused controversy in the field of gifted education has been: 'How can we identify a gifted student from his/her peers?'Gagné was particularly concerned about treating gifted students as belonging to a homogenous group arguing that there are different levels of giftedness. As an intrinsic component of his model, Gagné then developed a clear and defensible metric based system (MBS) whose conceptualization posits a five-level system of cut-offs for giftedness as follows: "mildly" 10% (top 1:10); "moderately" 1% (top 1:100); "highly" 0.1% (top 1:1000); "exceptionally" 0.01% (top 1:10,000); "extremely" 0.001% (top 1:100,000). Using this MB She argued that the mildly gifted (1:10) or the top 3 achievers in a regular class of 30 already distance themselves very significantly in terms of ease and speed of learning. He referred to such mildly gifted students as the 'garden variety'—a common English expression in the USA that means the 'most common group'. Similarly Renzulli (2012) used the terms 'high achieving' or 'schoolhouse giftedness' to refer to students who are good lesson learners in the traditional school environment. So in this paper the term 'mildly gifted' is used in accordance with the recommendations of Gagné (2015), Renzulli (2012), and Shayshon et al. (2014), to refer to 1:10 students who attend everyday regular class and who demonstrate relatively high mathematical ability.

The focus on these 'mildly gifted students' follows Gagné's recommendation that the vast majority (90%) of the gifted/talented individuals belong to this lowest level while the highly gifted/talented (1:100,000) individuals are a rarity. The level of this rarity is such that even full-time teachers of the gifted, in the course of their 35-year professional careers, may encounter just a few if any of these extremely gifted students. His concern was that when we present extreme examples of behaviour to parents or teachers, we risk conveying a distorted image of the 'garden variety' of gifted individuals because stakeholders would be tempted to judge that such a rare population does not justify large investments of time and money to meet their educational needs. Gagné (2010) therefore recommended that gifted and talented program coordinators should think first and foremost about services for their mildly gifted students. In education systems that are guided by the inclusive philosophy, the 'garden-variety' of gifted and talented students spend the majority of their time in regular classrooms hence it can be argued that every teacher should be regarded as a teacher of the gifted and talented. Given its heuristic appeal and elegance, for the purposes of this paper I found Gagné's model particularly relevant as a rudder and compass to frame the arguments.

1.2 Conceptualisation of different representations

Given that one of the research questions is about teachers' representational fluency, it is important to show how this idea has been conceptualised. In the literature, this ability is sometimes referred to as representational flexibility, representational fluency or representational thinking (Pape and Tchoshanov 2001). Regardless of the term used, each emphasizes the value of students' ability to work proficiently with varied representations and how that ability supports students' success in learning mathematics. So in this paper representational competence in mathematics is viewed as the ability to comprehend the equivalence of different modes of representation as well as to use such modes of representation meaningfully for the purpose of communicating mathematical ideas and solving problems (Sigel and Cocking 1977). In the mathematics education community, these modes of representation have been based on different theoretical perspectives. Lesh, Post and Behr (1987), for example, pointed to five representations including real world object representation, concrete representation, arithmetic symbol representation, spoken-language representation and picture or graphic representation. While each of these different types of representations can be studied separately Gagatsis and Shiakalli (2004) provided a more comprehensive lens that encompasses all. They suggested two major registers of representations, namely, treatments and conversions, which Businskas (2008) identified as equivalent and alternate representations respectively. Representations are equivalent/treatments if they are in the same register and they are alternate/conversions if they are from different registers. For example the graph of a parabola is an alternate representation/conversion of $f(x) = ax^2 + bx + c$ because the two representations are from two different registers (graphic—symbolic). On the other hand 13 + 23 is equivalent to 23 + 13 is equivalent to 36 or $f(x) = ax^2 + bx + c$ is equivalent to $f(x) = a(x - p)^2 + q$. In these examples, both the first and second form are symbolic. Within the literature the importance of learners being able to move comfortably between and among, within and across, these multiple representations is highlighted. This significance suggests that both the alternate and equivalent representations should be developed. Consistent with this bifocal perspective, the term different representations is used in this paper in relation to both treatments and conversions. Classroom activities were therefore analysed to see the extent to which the teachers' representational fluency between and across different registers enabled or constrained learners' creativity.

Literature however cautions that not all representations are beneficial for creativity to flourish (Mhlolo et al. 2012). So the next question could be 'How can we begin to judge the quality of the different representations that teachers promote in a classroom situation? Martin and Schwartz (2014) offered a cognitive analysis of how visual representations can create opportunities for creativity, whilst also considering the ways in which they might hinder it. Key to their analysis is the view that the ways in which mathematical validity is established within mathematics classrooms are important. Yackel and Hanna (2003) describe the process as giving reasons for a mathematical action or statement in an attempt to communicate the legitimacy of one's mathematical activity. Similarly Sierpinska (1996) argued that acts of deep understanding that promote creativity link what one must understand with the basis or reasoning for that understanding. Andrews (2009) gave a bifocal view (teacher and learner) when he argued that teachers' representations which are accompanied by (a) articulation, justification and argumentation from the teachers and/or (b) pressing for reasons why from the learners, could lead learners into developing their creative potential. In this paper I argue that teachers' representations positively impact students' creativity only if the representations are mathematically precise and accompanied by further articulation or reasons why. I was therefore looking at the quality of teachers' different representations between and across different registers and analysing the way teachers provided mathematically valid statements, provided justification (reasons why) for their statements and also pressed for reasons why from their learners.

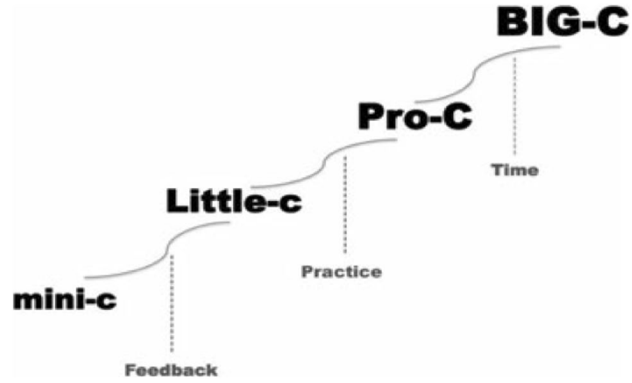


Fig. 1 The developmental trajectory of creativity (Beghetto 2014)

1.3 Conceptualisation of creativity

Although mathematical creativity has been described as the most important economic resource of the 21st century, there is lack of consensus on its definition (Mann 2006). Currently most investigations of creativity tend to take one of two dichotomous directions, i.e., the Big-C/little c creativity. Although Big-C creativity is clearly defined, Beghetto and Kaufman (2007) argued that the Big-C/little-c debates rest on a false dichotomy that obscures the blended nature of creativity and proposed that the time has come to consider how the Big-C/little-c conceptual framework can (and should) be broadened. In their four C's model (Fig. 1) Kaufman and Beghetto (2009) proposed two additional categories 'mini-c' to encompass initial creative interpretations and 'Pro-c' as an appropriate category for individuals who are professional creators, who have gone beyond little-c but have not reached the eminent status of Big-C. Beghetto and Kaufman (2007) proposed the concept of mini-c not simply to create another framework of creativity, but they argued that everyone is creative, and that this creativity all starts in the mini-c, which in most cases can become little-c; in extra-ordinary cases little-c may then turn into Pro-c or Big-C but in other instances mini-c might never evolve. The proposal made by Beghetto and Kaufman (2007) was not an arbitrary classification but was based on solid empirical evidence (e.g., Baer and Kaufman 2005; Cohen 1989; Sawyer et al. 2003).

According to Beghetto and Kaufman (2014) the Four C's model can help teachers understand the levels of creative expression most germane to the classroom environment (i.e. mini-c and little-c) and identify key factors necessary for supporting the development of creativity from one level to the next. Beghetto and Kaufman (2007) argued that having a further sub-division of little-c (into mini-c) creativity helps to highlight the importance of considering the developmental nature of creativity. The inclusion of mini-c creativity was perceived as offering an additional unit of analysis for creativity researchers interested in studying the creative potential and development of children and novices. Inclusion of the mini-c category becomes clear when we consider the standards used to judge the creative insights of elementary or high school students (Kaufman and Beghetto 2009). In this case mini-c creativity represents the initial, creative interpretations that all creators have, which later manifest into recognizable (and in some instances, historically celebrated) creations. Mini-c creativity seems particularly suitable for the educational sector, where the priority is to encourage all students and pupils, who have not yet reached their intellectual peak, to achieve their full potential. According to this idea, creative potential can be found in every child (Runco, 2003); it can be encouraged or inhibited (Sharp, 2004); and its development depends on the kind of training people receive (Esquivel 1995). This conceptualization of mini-c creativity is important in this paper given the paper's focus on the support given by mathematics teachers to the 'mildly gifted' students. I argue that mildly gifted students are likely to exhibit such min-c creativity.

1.4 Teacher responses that support/inhibit the mini-c creativity

With reference to teachers' recognition and support of students' creativity, Beghetto (2013) posits that a potentially creative idea may first appear as an unexpected idea (micro-moment) which warrants some level of recognition and exploration by teachers. Beghetto defines micro-moments as brief, surprising moments of creative potential that emerge in everyday routines, habits and planned experiences. In these micro-moments, the students break from the normal set of responses to mathematical tasks and look at the mundane through a new set of eyes. When students respond in unexpected ways to known answer questions, teachers are then confronted with micro-moments decisions (Beghetto 2013). Given that creativity is a distinguishing characteristic of giftedness, the way teachers respond in these micro-moments has important implications for whether opportunities for nurturing mini-c creativity will be supported or missed. Although there are a variety of ways teachers can respond to such student creativity, Rowland and Zazkis (2013) suggest that the teacher's response is one of three kinds, namely, to ignore, to acknowledge but put aside, and to acknowledge and incorporate. While acknowledging an idea and putting it aside for some time might not have significant consequences, there are costs and benefits inherent in choosing between the two extreme responses i.e. 'ignoring' or 'acknowledging and incorporating' students' unexpected responses. In order to examine the extent to which the teacher's acceptance/dismissal of unexpected situations is of value or of no value to the students, two important factors need to be considered (a) the (non)mathematical nature of the student's contribution and (b) the (non) mathematical nature of the concomitant teacher's response.

2 Methodology

2.1 Research design

This paper draws from archived data collected from a large-scale research project in which the researcher prioritised the challenges faced by mathematics teachers in implementing curriculum change in previously disadvantaged communities. The decision to re-visit the archived data is consistent with what Irwin (2013) defined as Qualitative Secondary Analysis (QSA) which refers to the (re) using of data produced on a previous occasion to glean new social scientific and/or methodological understandings. This may involve prioritising a concept or issue that was present in the original data but was not the analytical focus at that time (Irwin 2013). In the present analysis the priority is given to gifted learners' creative potential that was present in the original data but which was evidently ignored previously when teachers were prioritised. Proponents of QSA contend that opportunities to ask new questions and so to draw new interpretations are some of the reasons for returning to one's own data or turning to the data of other researchers (Bornat 2010). Similarly Mason (2007) suggested that we can come to understand re-using qualitative data not as the re-use of pre-existing data, but as a new process of re-contextualising data leading to the generation of new knowledge.

2.2 Participants

Four Grade 11 mathematics teachers (2 male, 2 female) with over ten years of experience were observed teaching in regular classrooms of ±40 students. In each of the four classrooms it was possible through the teacher's nomination and book inspection to identify four top performers who in Gagné's definition would constitute the 'garden variety' of intellectually gifted students. In this paper I

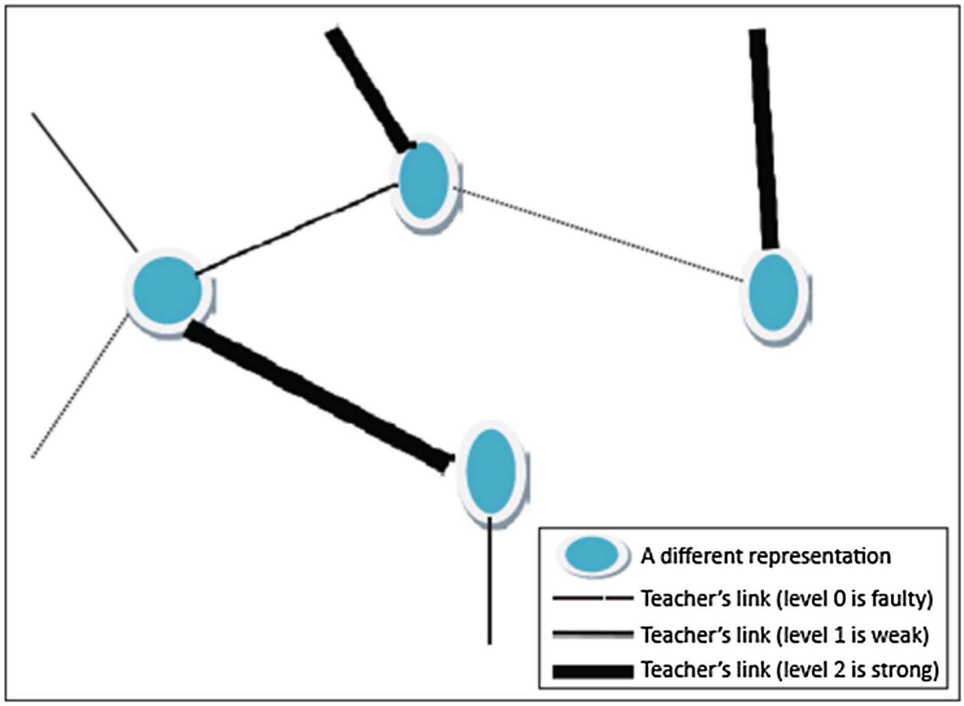


Fig. 2 Representational reasoning-model (Mhlolo et al. 2012, p.4)

prioritise these 16 students' ways of creative thinking and how their teachers supported or hindered their progress.

2.3 Data analysis techniques

In order to answer the first research question, I analysed transcripts of 20 lessons and coded 377 teachers' different representations. Coding of teachers' mathematical representations was done in accordance with a Representational Reasoning Model (Mhlolo et al. 2012) that we developed—see Fig. 2.

For example a teacher's utterance was coded as follows:

DR 0 (*dotted line*) if it was a different representation that was mathematically faulty,

DR 1 (*continuous but light line*) when the different representation was mathematically correct but superficial or routinely algorithmic, with no further explanation or justification; then.

DR 2 (*continuous bold line*) when the different representation was more than just mathematically correct but where justification and/or further explanation followed.

Table 1 provides a few examples of the teachers' different representations and how these codes were applied.

2.3.1 Comment

For example Teacher B's lessons for the week were on the topic 'Functions and Algebra' and focused on Calculus with specific interest on gradient of both straight lines and curved lines. The first column shows what the teacher was saying, column 2 exemplifies how data were categorised into each of the levels of quality of the different representations, and column 3 shows the researcher's comments. In the first episode we see a different representation in the sense that the teacher represents zero gradient graphically by drawing a horizontal line, then verbally—she calls it a horizontal gradient. The episode was then coded DR0 because it is mathematically faulty to describe a horizontal line as having a horizontal gradient. In the second episode the teacher again shows a different representation in the sense that the value of m in a standard equation for a straight line $y = mx + c$ stands for the gradient. However the different representation was considered to be mathematically correct but superficial or routinely algorithmic given that the teacher does not seem to explain clearly what it means to have an increasing or decreasing gradient. The episode was then coded DR1—correct but with no further explanation or justification. The last episode again shows a different representation in the sense that from a symbolic/algebraic representation $f(x) = x^2 + 1$ the teacher generates a table of values. The teacher explains well how the x values or inputs are substituted into the function machine to get the y values or outputs. The episode was then coded DR2.

In order to answer the second research question a classroom micro-moment was chosen as the unit of analysis. A micro-moment emerges when a student presents an unexpected idea. In coding micromoments the focus was on teachers' utterances in response to students' unexpected responses. The legitimacy of this approach can be

Table 1 Teacher R's excerpts showing different representations

Episode	Code	Comment
And remember here we are talking of gradient of a line. Okay. So be it, this is what It's horizontal, (and teacher draws a horizontal line)	DR0	A horizontal line is being defined as having a horizontal gradient
So if you have a horizontal line what does it tell you about the gradient? We hare a horizontal gradient.		
Look at the gradient of aaaa… let's say $y = 2x + 1$ the gradient is what its 2 okay a positive 2. It's a value that is greater than what than 0 okay. So the gradient will be increasing okay and if it was $y = -2x + 1$. Haa. It's negative so it's what its decreasing Okay. So the gradient of a line when, you talk of a gradient of a line given an equation of a straight line, okay a linear equation the coefficient of x is what is the gradient, Okay. So the sign before the coefficient is the one that tells you the gradient is what positive or it's what negative	DR1	Here the teacher is able to identify' the gradient correctly as the value of m in a standard equation for a straight line i.e. $y = mx + c$ but what does it mean to say when this m-value is positive it is increasing and when it is negative it is decreasing?
You can either do it in a table form whereby you have your input and your output. So you have what x as the input and then y as the as the output, We are going to substitute the x values into the function and then we will get what the y values, So we have $f(x) = x^2 + 1$	DR2	Another form of representing a function which appeared to be well explained
x −2 −1 0 1 2 *y* 5 2 1 2 5		
(The teacher then completes the table of values after which she raises another question)		

evidenced in many studies on scaffolding dating back to 1975. For example Wood et al. (1976) defined scaffolding as the process that enables a child or novice to solve a problem, carry out a task, or achieve a goal which would be beyond his unassisted efforts. More recently Bakker et al. (2015) devoted a special issue (volume 47 issue 7 with 22 articles) of the *ZDM International Journal of Mathematics Education* to studies which focused on scaffolding and dialogic teaching in mathematics education. Common in these studies on scaffolding is the evaluation of classroom interactions with respect to teachers' adaptations to students' needs. In this paper I argue that a student's unexpected idea might be mathematical (creative) or non-mathematical (disruptive) and the teacher might respond (adaptation) by ignoring or by acknowledging and incorporating. This then gives rise to the following possibilities and codes:

MAI = student's unexpected idea is **m**athematical and the teacher **a**cknowledges and **i**ncorporates it.

MIG = student's unexpected idea is **m**athematical and the teacher **ig**nores it.

NMAI = student's unexpected idea is **n**on-**m**athematical and the teacher **a**cknowledges and **i**ncorporates it.

NMIG = student's unexpected idea is **n**on-**m**athematical and the teacher **ig**nores it.

Due to space limitations I give an overview of just two micro-moments showing how students' unexpected ideas were mathematically sound but teachers ignored them (MIG). The decision to highlight these examples follows literature which shows that when teachers view creative ideas as disruptive and habitually dismiss them (MIG), they are seriously undermining opportunities for students to share and develop potentially creative ideas. Responding to a mathematical interruption non-mathematically might therefore be regarded as a missed opportunity and this is typical of a developmental process that is poor in Gagné's model. In such an environment gifted students will not develop to their full potential.

2.3.2 Micromoment 1

Teacher B's lessons for the week were on multiplying binomials and trinomials. We pick up the discussion when the teacher puts up five tasks on the board and asks five students to come and show how they would work them out. [*Notice that Senzo and Brilliant (pseudonyms) are the mildly gifted students here].*

(a) $(b-4)(b^2-4b+16)$ (b) $(a+b)(a^2-ab+b^2)$
(c) $(x-y)(x^2+xy+y^2)$ (d) $(a-1)(2a^2+a+1)$
(e) $(3x^2+xy-2y^2)(x+2y)$

Because task (e) was arranged differently (trinomial to the left binomial to the right) from the other four the teacher chose Senzo [the clever one] to work it out. Below Senzo explains her working as follows:

Senzo: $(3x^2+xy-2y^2)(x+2y)$.
$(3x^3+6x^2y+x^2y+2xy^2-2xy^2-4y^3)$.
$(3x^3+7x^2y-4y^3)$.

I notice that that there is no other groups of x^3 and so this comes down [pointing to the $3x^3$ coming down into the final answer section]. These two $+6x^2y + x^2y$ add up to $+ 7x^2y$. The $+2xy^2$ and $-2xy^2$ they cancel out because of the signs. The $-4y^3$ remains as it is and so this is the final answer. ($3x^3 + 7x^2y - 4y^3$).

Teacher: What are you saying about her approach? How did she approach this? She was finding the product of binomials and trinomials using the distributive law. Did she apply the distributive law?

Class: No.

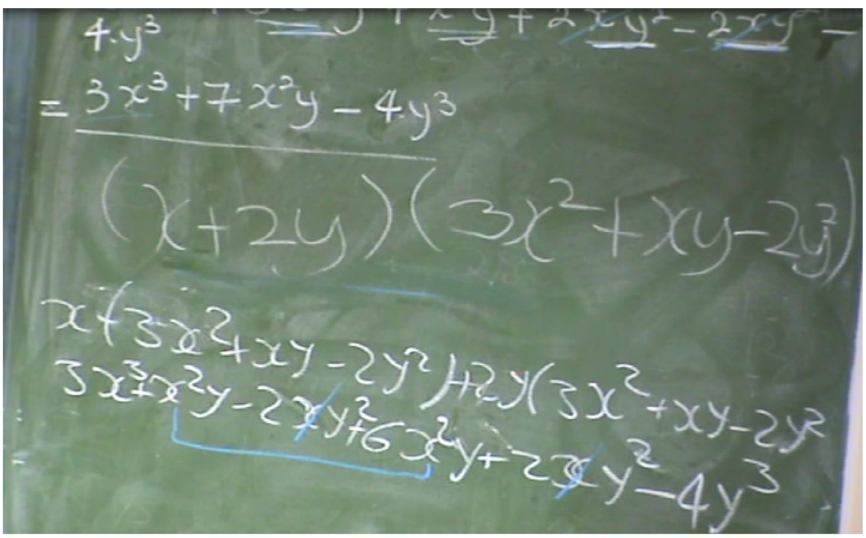
Teacher: They emphasise here in brackets [pointing to the textbook] that apply the distributive law. What was she supposed to do first before she multiplied? Can someone come and correct her. We don't want to erase her work we just want to correct her as it is.

Learner 1 [comes to the board and starts by re-arranging putting the binomial on the left then works the task as shown on this clip and gets the same result $3x^3 + 7x^2y - 4y^3$ as Senzo]

there are those finer lines in a question that say we can get the same answer but if it was in an exam I was not going to credit her because she did not follow instructions from the question which is very important. Do you understand me?

2.3.3 Comment

We can possibly see that Senzo's way of finding the product of a binomial and trinomial was unexpected hence the teacher drew the attention of the class to this student's approach. However this approach is mathematically meaningful and therefore creative in a number of ways. This learner appeared to recognise equivalent representation through the commutative law hence she saw no need to re-arrange starting with the binomial on the left hand side. The approach is efficient (fluent) in that it quickly brings the like terms closer to each other hence in just three steps the student was able to get to the solution. The teacher however ignored it completely suggesting that the student



Senzo: There is the answer Ma'am

Teacher: Yes, what we said was the answer is the same but the approach was different. So next time you should read the question because the question says apply the distributive law. Ok.

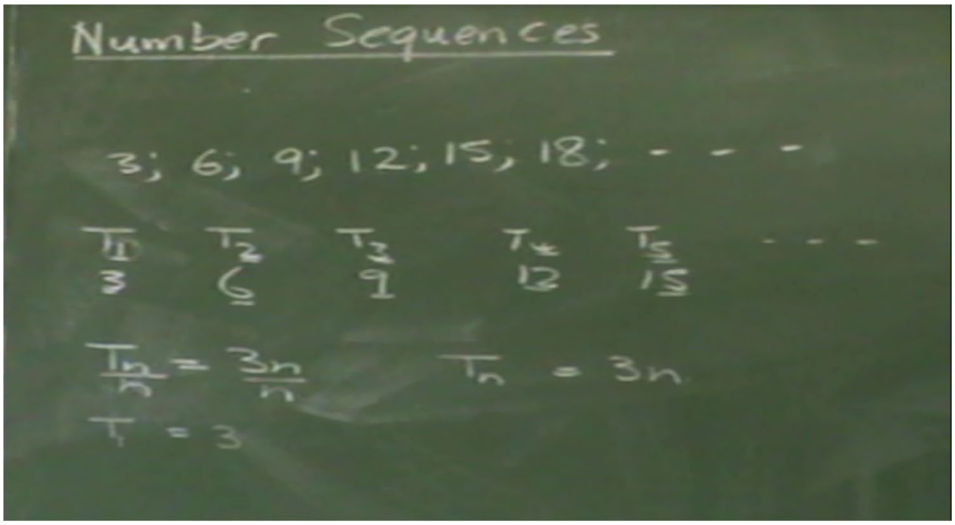
Brilliant: [pointing to Senzo's work] But Ma'am I understand her approach better.

Teacher: We are following instructions. Ok, ok if it was just ordinarily finding the product of binomials and trinomials really she was correct. But now in brackets

was not following instructions and that if it was in the exam she was not going to credit it. The micromoment was coded accordingly as **[MIG]**.

2.3.4 Micromoment 2

Teacher M's lessons for the week were on number sequences. We pick the discussion when the class was trying to find the general term for a linear sequence [*notice that Hazel ((pseudonym) is the mildly gifted student here]*



Teacher: Alright, I want us to observe a pattern here. Term number 1 is 3. What has been done to this 1 to make 3; the same thing should be done to this 2 to make 6; the same thing should be done to this 3 to make 9 and so on. So what will T_n be?

Learner 1: $T_n = 3n$ sir.

Teacher: Explain how you got that.

Learner 1: I just looked at the difference between the consecutive terms sir and multiplied by n.

Teacher: Ok. Somebody has made an observation here. To get the general term you simply look at the differences here [pointing to two consecutive terms] then multiply by the term number. But does it always work, let's see. (Teacher tries to generate all the terms in the sequence using this general term and the class agrees it works. He then puts another sequence on the board as follows)Teacher: What would be T_n in this case here?

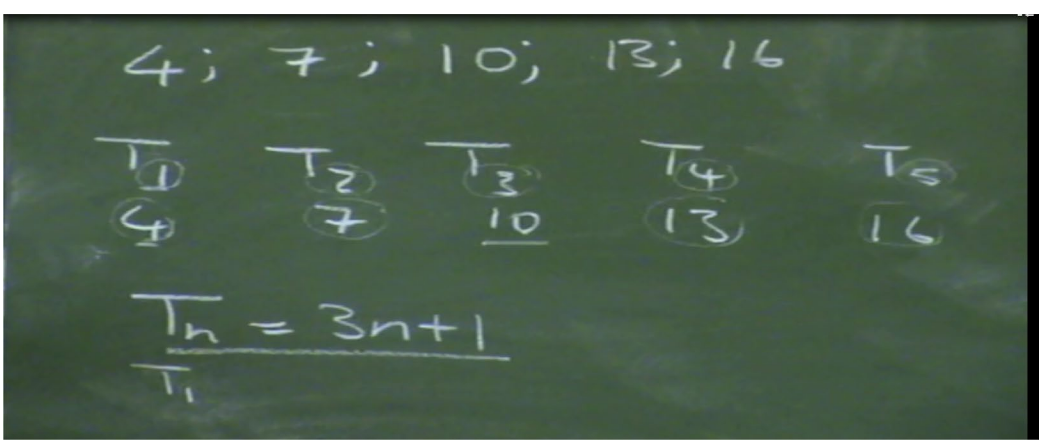
Learner 2: $T_n = 3n + 1$

Teacher: Explain how you get it.

Learner 2: It's the same. I looked for the difference between the consecutive terms and to get the +1, I simply said; what must I do to this 3 to get my first term in the sequence.

Teacher: Again somebody has made an observation here. To get the general term you simply look at the differences here (pointing to two consecutive terms) and to get this c in the general term you simply say what is the difference between this 3 and the first term 4. Anybody who does not agree with that?

Teacher: [Writes on the board 3; 7; 11; 15; ….] alright, you are given these four terms of the sequence; the general term?



Learner 3: $T_n = 4n - 1$.

Hazel: Can I please ask a question? You see I just want to find out why isn't that to find the T_n = blab la bla why can't you just add those numbers I mean for example like three (T_1) then you add one, two, three, four [constant difference] then you put the four instead of getting all the other things for the T_n.

Teacher: Ok you can start afresh. What are you saying? What are you suggesting?

Hazel: Sir why can't we just like find the differences?

Teacher: We find the differences fine, like in this case the difference is what? It's four.

Hazel: Yaa its four

Teacher: It is four

Hazel: Yaa. Then why is it that you can't write like T_n = bla bla bla + 4? Why do you have to write -1 that's my question?

Teacher: Right the general term is some kind of a formula that will be used to generate all the terms of the sequence. It's ok.

Hazel: Yes, yes

Teacher: Right, can you say $T_n = 4$ is a formula?

Class: Nooooooo

Teacher: [to Hazel] Ok, alright I thought you had made an observation.

Hazel: Sir I do have an observation.

Teacher: Ok order (the class is making noise). Ok let's give somebody else a chance. [Hazel is then ignored and the lesson continues]

2.3.5 Comment

In the last example with a constant difference of 4 Hazel was asking why +4 was not coming out in the general term. The fact that she was persistently asking this question clearly indicates that she was thinking in the recursive. With specific reference to the generation of a recursive or explicit rule for a sequence, Blanton (2008) cautions that it is important to listen to how learners' verbal statements imply that they are looking at the ways that quantities change (recursive), or that they are making a prediction based on the connection between the term number and its value (an explicit general term). Recursive reasoning emerges naturally, as learners develop skip counting and the ability to add on. Recursive reasoning is therefore seen as a building block for the eventual ability to use formulae that directly determine any unknown amount. For learners in the early stages, instruction that encourages them to look for recursive patterns in functional situations is a recommended starting point for developing algebraic thinking (Bezuszka and Kenney 2008). In all the examples worked here the teacher's direct link between the term-numbers T_n and their values to obtain the general term therefore masked the recursiveness of the constant difference. So while the question by Hazel was not expected, there was logical mathematical sense in the student's question because recursive reasoning emerges naturally. This suggests that there was potential for students to make sense of the explicit rule through the recursive rule. The teacher however ignored it again reprimanding Hazel that '*I thought you had made an observation*'. Although Hazel insisted that she had an observation, the teacher calls for order and asks for another student to make a contribution. The micromoment was also coded as **[MIG]**.

2.4 Validity and reliability

A number of measures were taken to enhance the accuracy, credibility and validity of data. Firstly participation was voluntary and there were frequent member checks with the participants. During the lesson observations there was constant dialogue with participants in order to verify the researcher's inferences. After the lessons participants were asked to read transcripts of dialogues in which they had participated in order to either agree or disagree that the summaries reflected their views, feelings and experiences. Micro-moments were coded by three different peers whose inter-rater reliability was 0.86. Throughout the research process the study was also subjected to peer scrutiny through conference presentations, peer discussions and research indabas (discussion groups).

3 Results

Research Question 1—To what extent does teacher representational fluency support/hinder gifted learners' creativity in the regular classroom?

Figure 3 provides a summary of the levels of these different representation episodes for all the four teachers for the whole week.

3.1 Comment

The results show that teachers' representations were mathematically faulty (level 0 = 140 times); correct but with no further justification or explanation (level 1 = 120 times) and correct with further justification (level 2 = 117 times). When one considers that level 2 of knowledge quality should be the target of any effective classroom teacher, then it can be argued that teachers were off that target in almost 70% cases of their different representations. This left only 30% of the different representations with the potential to support gifted students creativity.

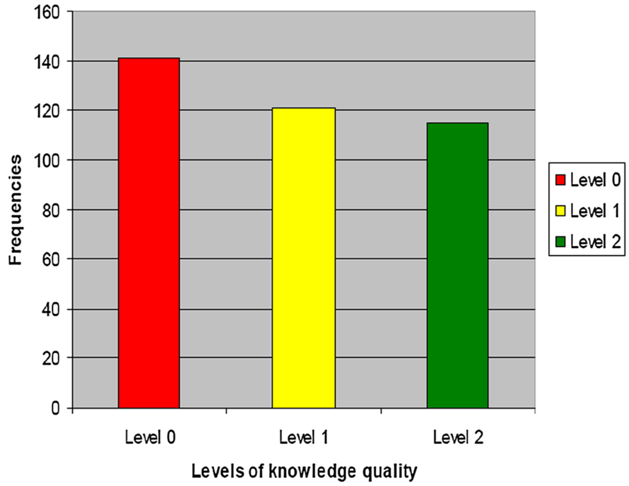


Fig. 3 Summary showing the quality of different representations (n = 377)

Research Question 2—In what way do gifted students demonstrate their mathematical creativity and how do teachers support or inhibit such creativity?

3.2 Comment

The results show that throughout the 20 lessons that were observed over a period of 4 weeks, a total of only 43 unexpected ideas came from the mildly gifted learners, 38 (89%) of which were mathematical/creative and only 5 (11%) were non-mathematical and therefore considered disruptive. This low prevalence of unexpected ideas is well justified in the literature given that all too often teachers may view this behaviour as the student's impertinence or criticism of their teaching methods (LaFauci and Richter 2000). Similarly Beghetto (2007) shows that there is a tendency among teachers to prefer standard answers to unique ones; as actual teaching culture does not value creative answers. Teachers therefore usually view unexpected ideas as disruptive and consequently discourage such behaviour in the interest of meeting curriculum expectations (Table 2).

4 Discussion

The first research question for this paper was about how teachers use different representations to create/hinder opportunities for gifted learners' creativity in the regular classroom. It would appear that gifted students in these regular classrooms lost opportunities to develop their creative potential given that only 30% of teachers' representations were mathematically accurate and supported with justification or further explanation and the majority (almost 70%) of teachers' representations were either faulty or superficial. Similar observations were made by Davis and Johnson (2007) who concluded that in South Africa teachers spent most of their time on explaining mathematical ideas, principles and definitions but without discussing or explicating the mathematical reasons for the productions of the ideas. All this seems consistent with other observations that in South Africa teachers' subject knowledge is weak (Adler 2009; Spaull 2013). Given that the instructional representations that gifted students encounter define the formal opportunities for learning about the subject content it can be argued that gifted students might not be developing to their creative potential under current conditions in South Africa. A recommendation that could be made follows from Ball and Bass (2003) who argued that effective teachers of mathematics have to use mathematically appropriate and comprehensible definitions, represent ideas carefully, mapping between a physical, graphical model, symbolic notation, and the operation or process.

The second research question had to do with the recognition of some of the creative abilities that are demonstrated by mathematically gifted learners in the regular classrooms. Generally the results show evidence of creative and productive thinking in gifted learners' contributions given that 89% of the students' unexpected ideas were mathematically reasonable in context. However such creative ideas were only acknowledged and incorporated into the teaching and learning in only 26% of the cases. In 63% of the cases such students' creative ideas were considered disruptive and were therefore ignored. This is despite the fact that in some cases such learner contributions had potential to open up opportunities for more conceptual than procedural understanding. For example in micromoment 2 we see how the teacher adopted a more procedural approach in developing the general term for a linear sequence. Using an example like, 3; 6; 9; 12…… as an entry point and asking the students to generate the general term ($T_n = 3n$) by linking the term numbers and their values in the sequence suggests a more procedural approach which masks the *c* value of the general term. On the other hand Hazel was

Table 2 Summary of micromoments (n = 43)

	Teachers responses	
Nature of students' unexpected ideas	Acknowledged and incorporated	Ignored
Mathematical	11 (26%)	27 (63%)
Non-mathematical	0 (0%)	5(11%)

suggesting the recursive route as an entry to understanding the explicit or general term for a linear sequence. This was likely to be more productive given that recursive reasoning brings out clearly the idea of the constant value of 3 being continuously added as the sequence grows from one term to the next. Yet the teacher dismissed the learner; '*I thought you had made an observation.*' This is inhibitive to the growth of students' creative potential and their conceptual understanding.

In some cases the teacher's weak content knowledge seems to be exposed and in the fear of moving into the unknown the decision is to dismiss the learner's creative ideas. In a study by Leikin et al. (2013) similar observations were also made that there was a relationship between teachers' creativity and the depth of their mathematical knowledge. For example in micromoment 1 it can be argued that Senzo displayed some mini-c creativity in her approach to multiplying binomials and trinomials. She showed a clear understanding of some of the laws of operations especially the commutative law. We know for example that no matter in what order multiplication is carried out, the product will always be the same i.e. ab = ba. In the example that Senzo worked out, it can be argued that the trinomial and the binomial can be multiplied in any order. This multiplication is therefore both left and right distributive and the results are logically equivalent. Senzo's approach was also efficient in that the like terms automatically come adjacent to each other so that they become relatively easy to identify and work with. This is typical evidence of fluency or an ability to select a procedure that would be efficient in solving a specific problem. But Senzo's approach was dismissed by the teacher presumably because the teacher's (mis)interpretation of the Distributive Law from the textbook was that the binomial must always be presented on the left hand sided before the trinomial. We see this when the teacher says: '*We are following instructions*'. Senzo was even reprimanded in the sense that if it was in an examination the teacher was not going to credit her because she did not follow instructions from the question. When such events happen, students learn that the goal of class discussion is not to try to work out their own interpretation or understanding, but rather attempt to puzzle-out or guess the answer expected by the teacher. Consequently classroom discussions become more like "intellectual hide-n-seek" (Beghetto 2007) than opportunities for students to express and develop their own personally meaningful understandings.

The results from this study are similar to some studies that suggest that teachers tend to place a low value on creativity traits as being useful in the school environment. Other studies have shown that creative ideas tend to "pop up" at any moment, often catching the teacher by surprise (Crutchfield 1993). All too frequently, teachers may view this behaviour as the student's impertinence or criticism of their teaching methods (LaFauci and Richter 2000). Kennedy (2005) summarised these fears by noting that some teachers frequently mentioned a fear of chaos, others a need to stick with the plan, others a personal need for order. These elements can be seen in the way teachers dismissed students' creative ideas in the micromoments that have been analysed with teachers saying; "…. *if it was in an exam I was not going to credit her because she did not follow instructions from the question (teacher R); I thought you had made an observation (teacher M)*". Similarly, Beghetto (2007) also found that unexpected student comments were generally viewed as less preferable and more likely to turn into potential distractions. Many teachers experience the same dilemma of wanting to incorporate creative learning activities into the classroom but feeling that doing so comes at the cost of students' academic subject matter learning. Despite these concerns gifted students require instructional and curricular adjustments that create a better match between their identified needs and the instructional services they typically receive. Opportunities must be presented to allow students to show these characteristics and if their needs are ignored this may lead to loss of motivation, thereby preventing such students from learning which in turn leads to their underachievement.

5 Conclusion

A number of studies specifically focusing on gifted education in South Africa have made similar indications that far too many of the gifted students currently do not stand even the remotest chance of achieving near their potential because they are not receiving adequate support within mainstream classrooms (Donohue and Bornman 2014; Kokot 2011; Oswald and de Villiers 2013). But how can this lack of support for creative minds be explained in the South African context? There is empirical evidence to show that creative behaviour in students is often perceived by teachers as associated with scepticism and egoistic manners. Therefore many regular classroom teachers find themselves feeling caught between the push to promote students' creative thinking skills and the pull to meet external curricular mandates, increased performance monitoring, and various other curricular constraints (Beghetto 2013). Teachers prefer learners who have characteristics that are in sharp contrast with creative personality traits, such as "conforming" and "considerate". The more creative a class becomes, the less desirable their behaviour appears to teachers given that a creative teacher loses an aura of authority. In spite of this dichotomous situation that teachers find themselves in, Baer and Garrett (2010) pointed out that teaching for creativity and teaching specific content knowledge (as stipulated in the curriculum) are not in opposition and teachers can successfully meet both accountability standards and promote creativity in their classrooms. Treffinger et al. (2013) indicated that teachers, who hold a belief that student creativity can and should be developed, can teach for creativity even if they face challenges and concerns related to the educational system. Therefore, teachers can develop student creativity in the mainstream classroom even without the expenditure of extra time or the introduction of a new curriculum. However further research is needed to understand how this result can be achievable in the contexts of meeting curriculum standards.

6 Implications

This study has both theoretical and practical implications. Theoretically teaching gifted students in the regular classroom should not be conceptualised as some special kind of teaching. Approaches that are recommended for teaching gifted students are good for both strong and weak students. Practically regular classrooms in South Africa might not be conducive to mathematical talent development for the gifted students because their potential in creative ideas go unexplored due to many factors some of which include teacher knowledge, curriculum expedience and exam pressure. Following the results presented in this paper, I recommend that all teachers should be trained in gifted education in a way similar to that in which they are trained in special education. Currently gifted education is not included in most teacher education programmes. Further studies should be carried out to understand more about how gifted students are catered for in the regular classroom, in order for inclusive education be truly inclusive and catering for the needs of all learners.

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