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Review of current methods for estimating areal reduction factors applied to South African design point rainfall and preliminary identification of new methods

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Design point rainfall estimates assume a uniform distribution of rainfall over a catchment, and hence are only representative for a limited area. For larger areas, areal reduction factors (ARFs) are used to convert design point rainfall depths/intensities to an average areal design rainfall depth/intensity for a catchment-specific critical storm duration and catchment area. This paper presents a review of ARF estimation methods used nationally and internationally, with comparisons of the South African methods in the C5 secondary drainage region using standard input variables. The comparison of different ARF estimation methods confirmed that the empirical methods adopted for general use in South Africa are based on a limited database of observed rainfall data and are used without local correction factors beyond their original developmental regions. This results in the characterisation of the actual rainfall process over a catchment, and translation into questionable design peak discharge estimates. Therefore, the ARFs in South Africa need to be re-investigated in the light of recent extreme flood events, utilising the longer periods of record and denser rain-gauge networks which are now available for analysis. The variation of ARFs with return period and with rainfall producing mechanisms also needs to be investigated. Updated ARFs developed and verified using local rainfall data will improve the accuracy of design hydrology for large catchments in South Africa when event-based rainfall-runoff deterministic methods are used.

INTRODUCTION

In flood hydrology, the practising engineer or hydrologist frequently needs to estimate catchment design rainfall, i.e. rainfall information derived from observed rainfall data which comprises a depth or intensity, and duration associated with a given return period (T) or annual exceedance probability (AEP) (Gericke & Du Plessis 2011). However, design point rainfall estimates assume uniform spatial rainfall in a catchment, and hence are only representative for a limited area. For larger areas, areal reduction factors (ARFs) are used to convert design point rainfall depths/intensities into an average areal design rainfall depth/intensity for a particular critical storm duration and catchment area (Alexander 2001).

ARFs are estimated using either empirical and/or analytical methods. In many countries, the current ARF approaches are mostly based on empirical methods, using either storm-centred or geographically-centred approaches. Studies into the large-scale estimation of ARFs have generally been limited to the United States of America (USA) (USWB 1957; 1958), the United Kingdom (UK) (NERC 1975) and Australia (Siriwardena & Weinmann 1996). Despite these studies, ARFs are still regarded as inconsistent in most cases, mainly as a consequence of the variation in predominant weather types, storm durations, seasonal factors and recurrence interval (Skaugen 1997; Asquith & Famiglietti 2000; Allen & DeGaetano 2005). Omolayo (1993) identified insufficient rain-gauge networks and a lack of short duration (sub-daily) rainfall data as the main reasons behind the limited research in this field and the inconsistent results.

According to Asquith and Famiglietti (2000), the storm-centred approaches have not seen widespread application, due to the difficult inclusion of multi-centred storms. Omolayo (1993) indicated that storm-centred approaches are not suitable for estimating areal design rainfall from design point rainfalls, since extreme design point rainfall

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Table 1 South African ARF estimation methods

Method	Approach	Reference
Van Wyk (1965)	Storm-centred (graphical)	**Figure 1** (Van Wyk 1965)
	Storm-centred (numerical)	**Equation 1** (Op ten Noort & Stephenson 1982)
Wiederhold (1969)	Storm-centred (graphical)	**Figure 2** (Wiederhold 1969)
	Storm-centred (numerical)	**Equation 3** (Op ten Noort & Stephenson 1982)
Alexander (1980)	Geographically-centred (graphical)	**Figure 4** (Alexander 1980)
	Geographically-centred (numerical)	**Equation 4** (Op ten Noort & Stephenson 1984)
Alexander (1990; 2001)	Geographically-centred (graphical)	**Figure 5** (Alexander 1990; 2001)
	Geographically-centred (numerical)	**Equation 5** (Alexander 1990; 2001)
	Geographically-centred (numerical)	**Equation 7** (Gericke & Du Plessis 2011)

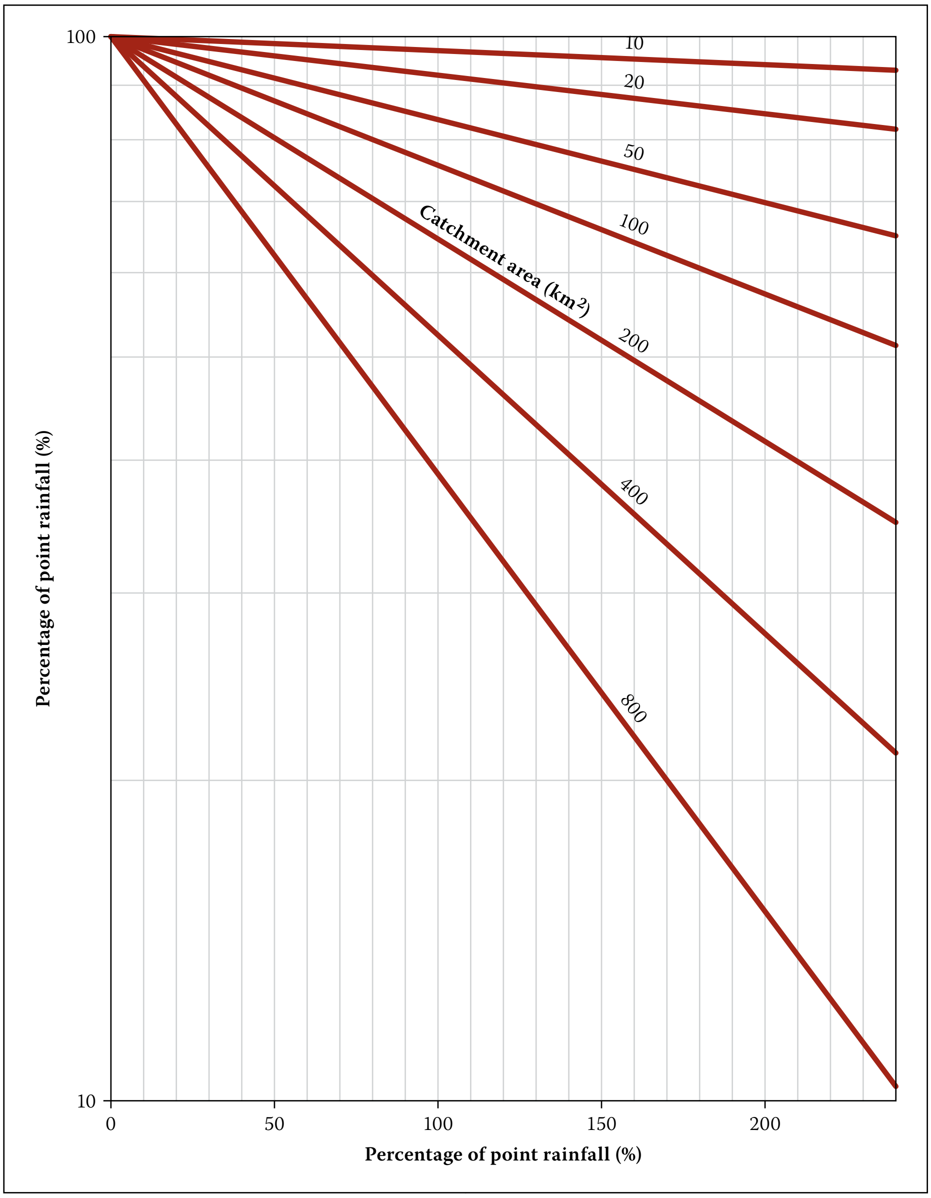

Figure 1 Expected percentage of runoff as a function of point rainfall intensity (SANRAL 2013)

and extreme areal design rainfall are unlikely to be produced by the same rainfall event or rainfall type.

The empirical methods used to estimate ARFs in South Africa are based on the limited research conducted using a storm-centred approach, e.g. Van Wyk (1965) and Wiederhold (1969), and a geographically-centred approach, e.g. Alexander (1980; 2001). The latter attempt is based on the United Kingdom Flood Studies Report (UK FSR) methodology (NERC 1975) using observed daily rainfall data up until the 1980s. There has also been a concern in the hydrological community in South Africa that the UK FSR results may not be appropriate to South African conditions.

During the last three decades, several new analytical methods have been proposed to estimate ARFs, e.g. storm movement (Bengtsson & Niemczynowicz 1986), crossing properties (Bacchi & Ranzi 1996), spatial correlation structure (Sivapalan and Blöschl 1998) and scaling relationships (De Michéle *et al* 2001). According to Svensson and Jones (2010), the level of agreement between the empirical and analytical methods currently in use is limited to a specific scaling regime, namely short storm durations and small catchment areas. Thus, these methods are regarded as inappropriate to use over a wide range of temporal and spatial scales, e.g. small catchment to a quaternary catchment level. On the other hand, a number of these empirical (storm-centred) and analytical (correlation-based and annual maxima-centred) methods do not provide probabilistically correct areal design rainfall estimates, as it is assumed that the AEP of both the point and areal rainfall is similar. Most of these methods are also based on a limited amount of observed rainfall data and use assumptions that are not entirely true descriptions of the actual rainfall process (Svensson & Jones 2010).

This paper provides preliminary insight into the applicability of the various methods used in South Africa and internationally to estimate ARFs. The objectives of the study reported in this paper are discussed in the next section, followed by an overview of the location and characteristics of the pilot study area. Thereafter, the methods used to estimate ARFs are reviewed. The methodologies involved in assessing the objectives are then expanded on in detail, followed by the results, discussion and conclusions.

OBJECTIVES OF STUDY

The objectives of this study are:

i. to review the ARF estimation methods currently used nationally and internationally, with emphasis on the inconsistencies introduced by the use of different approaches;
ii. to assess a selection of graphical and numerical ARF estimation methods used in South Africa, using standard input variables, e.g. catchment area, critical storm duration and rainfall intensity, in order to provide preliminary insight into the consistency between methods; and
iii. to compare the results from applying the selected ARF estimation methods using the C5 secondary drainage region of South Africa as a pilot study in order to

confirm or to reject the results as established in (ii).

In this study it was hypothesised that:

i. The graphical ARF estimation methods used in South Africa, as listed in Table 1, need to be updated with the longer data sets currently available compared to data used in their development, and these updates may influence the accuracy of the resulting average areal design rainfall and subsequent peak discharges as estimated with different design flood estimation methods.
ii. Most of the numerical ARF estimation methods used in South Africa are probabilistically incorrect, since these methods were derived from the original graphical ARF estimation methods, and possible differences between the recurrence intervals of point and areal design rainfall are not accounted for. In addition, the variation of ARFs with recurrence interval and rainfall-producing mechanisms is also not clearly understood. However, numerical ARF estimation methods are frequently used in practice due to the ease of calculation of the ARF, irrespective of the possible errors that could be introduced in doing so.

The recent compilation of the South African National Roads Agency Limited (SANRAL) Drainage Manual (SANRAL 2013), which is regarded by many practising engineers as an authoritative reference document, still proposes the use of these 'outdated' ARF estimation methods. This recent publication, along with the shortcomings identified during the literature review conducted on ARFs, served as a further motivation for this study.

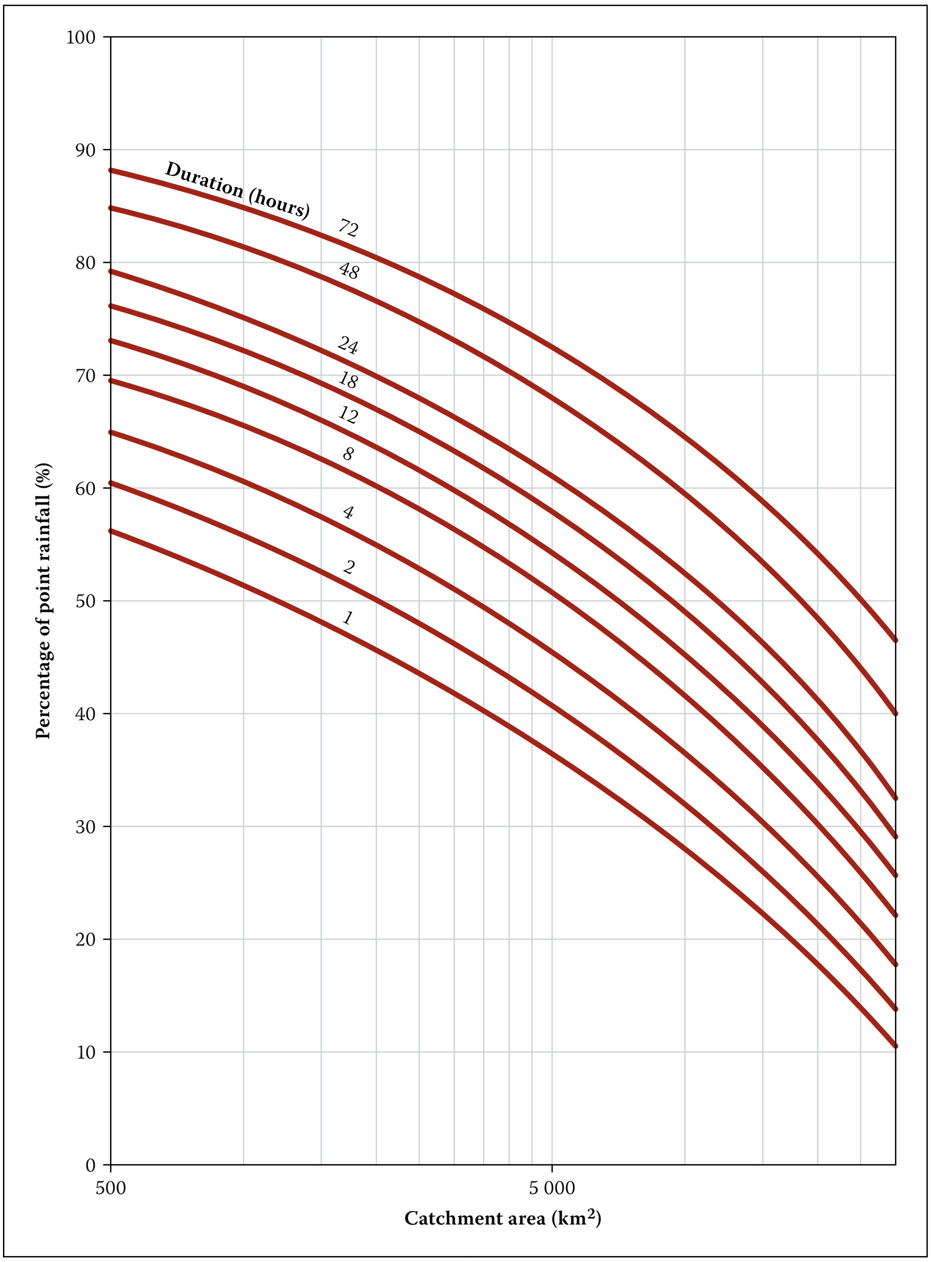

Figure 2 Expected percentage of runoff as a function of storm duration (SANRAL 2013)

REVIEW OF SOUTH AFRICAN ARF ESTIMATION METHODS

The following subsections provide a review of the storm-centred and geographically-centred empirical ARF estimation methods currently used in South Africa.

Storm-centred ARF estimation methods

Van Wyk's method (1965)

The first South African attempt to analyse ARFs based on a storm-centred approach was conducted by Van Wyk (1965; cited by Lambourne & Stephenson 1986) on a small scale (catchment areas ≤ 800 km^2) in the Pretoria region, Gauteng. In addition, a few rainfall storm areas from the USA and Canada were also analysed for comparison purposes. Isohyetal maps of several storms were plotted based on the average areal rainfall depths in catchments ranging from 10 km^2 to 800 km^2 centred on the maximum point rainfall and expressed as a percentage of point rainfall at the storm centre. The ARFs were also expressed as a function of the point source rainfall intensity, i.e. an average intensity over the storm duration at the storm centre. As a result, depth-intensity-area envelope diagrams (Figure 1) were developed, as included in the Drainage Manual (SANRAL 2013). In small catchment areas (≤ 800 km^2) the ARF is mainly a function of the area and design point rainfall intensity, since the relationship between rainfall intensity and the infiltration rate of the soil is predominant (Alexander 2001; SANRAL 2013).

Op ten Noort & Stephenson (1982) converted the ARF diagrams in Figure 1 to a mathematical algorithm (Equation 1) using regression analysis.

$$ARF = Exp(-0.000068\, iA^{0.77}) \qquad (1)$$

where:

ARF = areal reduction factor for point rainfall (fraction),
A = catchment area (km^2), and
i = point rainfall intensity at the storm centre (mm.h^{-1}).

Wiederhold's method (1969)

In the late 1960s, Wiederhold (1969; cited by Lambourne & Stephenson 1986) used a variable location, storm-centred approach, i.e. a modified version of Van Wyk's (1965) method to establish ARFs for 170 storms over large catchment areas between 500 km^2 and 30 000 km^2 within 18 regions delineated for South Africa. In these medium to large catchment areas (≤ 30 000 km^2), the ARF is mainly a function of the area and storm duration, since the quantity of rainfall relative to the number of storage areas is of great importance (Alexander 2001; SANRAL 2013). The large area storms were delineated, while

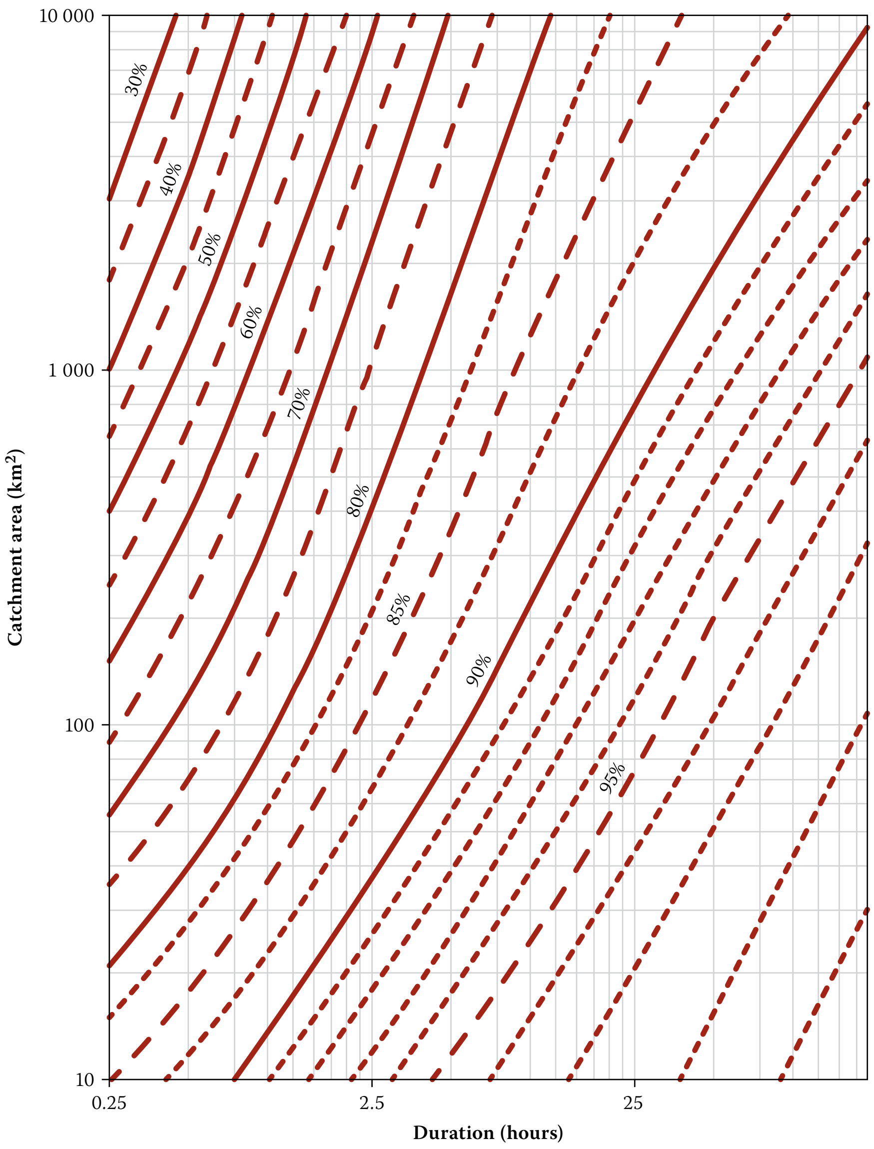

Figure 3 UK FSR ARF diagram (NERC 1975)

the point rainfall depths at each rainfall station were used to fit a sixth-order polynomial surface to enable the plotting of isohyets. Regionalised depth-area curves were produced for each storm with a time resolution of one day, resulting in co-axial diagrams to estimate the rainfall equalled or exceeded for storm durations of one day or longer. In the case of large area storms with associated storm durations less than 24 hours, the areal average rainfall over increasing areas (durations of one to six days) within each of the 18 regions were expressed as percentages of the maximum point rainfall observed. Depth-area curves were produced for durations of one to six days. Conservative upper envelope curves (of individual durations) were then re-plotted to produce depth-duration-area curves. Thereafter, the 24 hour to one hour durations were linearly extrapolated through these points to express the rainfall associated with a given area as a proportion of the point rainfall between one and 72 hours (Lambourne & Stephenson 1986). As a result, depth-duration-area ARF diagrams (Figure 2) for short to medium duration and large area storms were developed as included in the Drainage Manual (SANRAL 2013).

Op ten Noort and Stephenson (1982) converted the ARF diagrams in Figure 2 to a mathematical algorithm (Equation 2) using regression analysis.

$$ARF = [1.343 - 0.09Ln(A)]T_d^{0.03A^{0.19}} \qquad (2)$$

where:
ARF = areal reduction factor for point intensity (fraction),
A = catchment area (km²), and
T_d = storm duration (hours).

Op ten Noort and Stephenson (1982) compared Equations 1 and 2 and established that the use thereof could cause a discontinuity in storm runoff estimation. Subsequently Figure 2 was extrapolated so that the ARFs approach unity at short durations. This relationship is expressed in Equation 3.

$$ARF = [1.04 - 0.08Ln(A)]T_d^{0.02A^{0.28}} \qquad (3)$$

where:
ARF = areal reduction factor for point intensity (fraction),
A = catchment area (km²), and
T_d = storm duration (hours).

Geographically-centred ARF estimation methods

Alexander's method (1980; 2001)

Alexander (1980) developed a geographically-centred ARF relationship based on the ARF diagrams (Figure 3) contained in the UK FSR (NERC 1975). Alexander (1980) claimed that the developed ARF diagram (Figure 4) should be used when uniform temporal and spatial rainfall distribution over a catchment is assumed.

Op ten Noort (1984; cited by Lambourne & Stephenson 1986) converted these ARF diagrams (Figure 4) to a mathematical algorithm (Equation 4) using regression analysis.

$$ARF = [1.306 - 0.0902Ln(A)] + Ln(T_d)[0.0161Ln(A) - 0.0498] \qquad (4)$$

where:
ARF = areal reduction factor (%),
A = catchment area (km²), and
T_d = storm duration (hours).

Figure 4 was adjusted to account for short duration rainfall over small catchment areas, which are mostly characterised by severe storm mechanisms producing very high intensity rainfall with cell core areas exceeding 10 km² and durations exceeding 10 minutes. Estimates of shorter duration rainfall based on extrapolation from longer durations are unreliable when viewed in the light of the storm mechanisms which produce high-intensity rainfall for durations less than 10 minutes (Alexander 1980). Alexander (1980) argued that there is little justification in assuming ARFs less than 100% in these area and duration regions; subsequently Figure 4 was adjusted accordingly to provide a set of geographically-centred ARF diagrams (Figure 5) which enable the user to estimate average catchment rainfall from point rainfall statistics (Alexander 1990; 2001).

Alexander (1990; 2001) also converted Figure 5 to a mathematical algorithm (Equation 5). Alexander (1990; 2001) noted that the use of both Equations 4 and 5 resulted in slightly more conservative results when compared to the original UK FSR and United States Weather Bureau (USWB 1958) values.

$$ARF = [90\,000 - 12\,800\,ln(A) + 9\,830\,ln(60T_C)]^{0.4} \qquad (5)$$

where:

ARF = areal reduction factor (%),
A = catchment area (km²), and
T_C = time of concentration/critical storm duration (hours).

A recent study confirmed that a relationship existed between the catchment area, T_C and ARFs (Gericke & Du Plessis 2011). The validity of Equation 5 was assessed by plotting the T_C within each catchment under consideration against the catchment area, after which a power-law curve fitted through the data points was superimposed on Figures 3 and 4 respectively. The fitted power-law relationship as expressed in Equation 6 provided a good indication of T_C associated with any catchment area under consideration. Equation 7 resulted from the substitution and simplification of Equation 6 into Equation 5.

$$T_C = 0.2284\,A^{0.596} \tag{6}$$

$$ARF = [-6944.3\,Ln(A) + 115\,731.9]^{0.4} \tag{7}$$

where:

ARF = areal reduction factor (%),
A = catchment area (km²), and
T_C = time of concentration (hours).

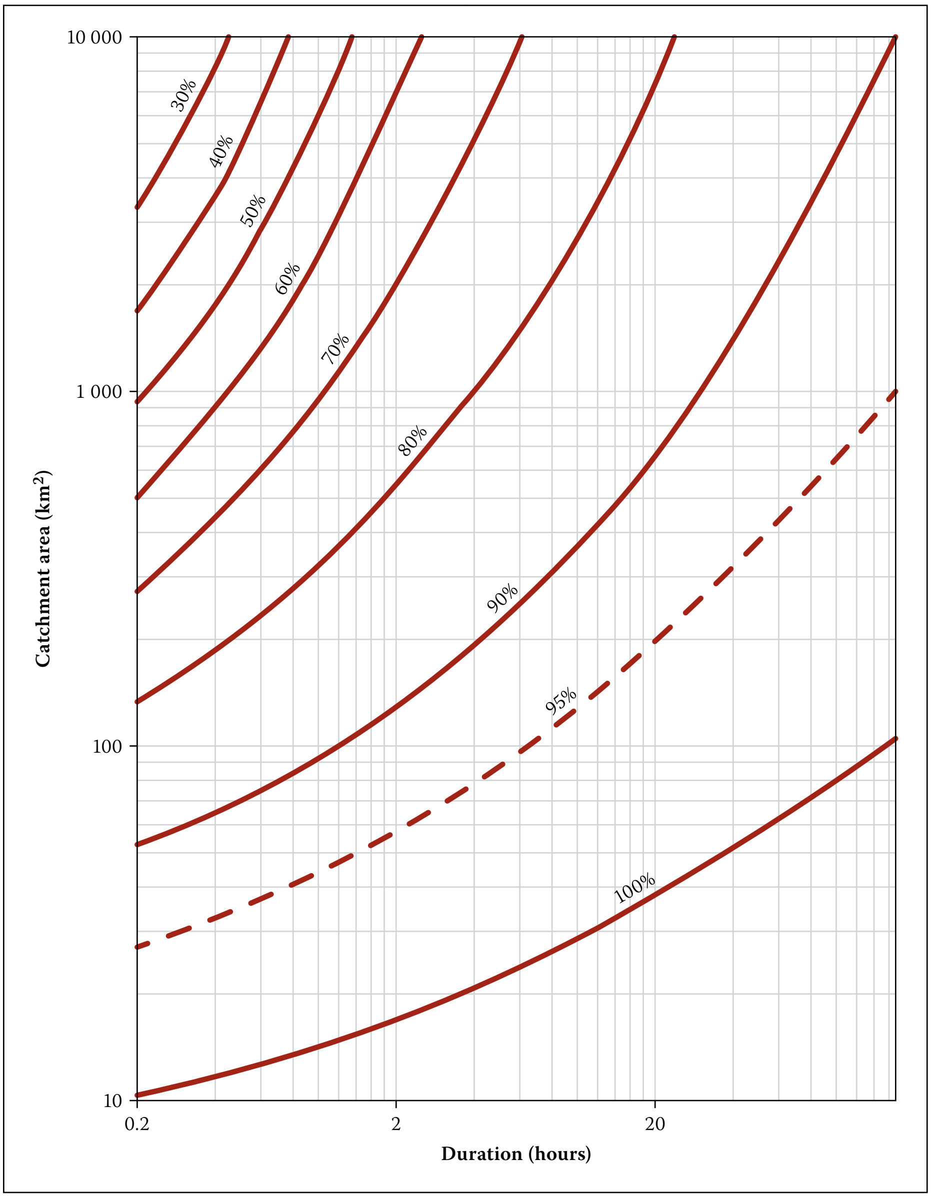

Figure 4 Adopted UK FSR ARF diagram for South Africa (Alexander 1980)

INTERNATIONAL ARF ESTIMATION METHODS RELATED TO THIS STUDY

The following subsections provide a brief review of the UK FSR (NERC 1975) ARF estimation method, which formed the fundamental basis of Alexander's ARF diagrams in the early 1980s, and Bell's method (1976). In this paper, the latter method is proposed as the most suitable for the re-investigation towards more up-to-date ARF estimation methods applicable to South African conditions. The justification of such a proposal is evident from the literature review to follow.

UK FSR method

The UK FSR method (NERC 1975) was developed by using nation-wide UK rainfall records. The final results from this study were represented using an ARF diagram (Figure 3), which allows the user to estimate geographically-centred ARF values based on a range of catchment areas and storm durations as input variables. Although this method allows for the area and duration to vary as input parameters, it remains invariant to the location within the UK and recurrence interval (Siriwardena & Weinmann 1996).

The method is reliant on the areal annual maximum event to recognise the date of the station/point annual maximum event. The point rainfall values corresponding to the same day as the areal maximum rainfall values (P'_{ij}) and the maximum point values at each rainfall station in the same year (P_{ij}) were recorded. This enabled the computation of ratios between P'_{ij} and P_{ij} at each rainfall station for each year. The final ARF values were derived from the overall mean values between the ratios for all stations and all years (Siriwardena & Weinmann 1996).

Bell's method

Bell (1976) conducted probabilistic rainfall analyses at rainfall stations (reasonably complete records over a 14 year period) situated in 1 000 km² circular catchment areas in the UK. The estimated frequency curves of areal and averaged point rainfall were used to establish ARFs, i.e. the ARFs were computed as the ratio of areal to average point rainfall with associated AEPs. A modified Thiessen weighting procedure was used to estimate the daily areal rainfall values, after which these values were ranked to obtain the 20 independent highest values for each sample area. In other words, a partial duration series (PDS) using equally ranked observations curtailed to a common base period were used and fitted to an exponential distribution, with parameters estimated by the Method of Maximum Likelihood (MML). The average point rainfall frequency curves were estimated using the 20 highest daily rainfall values at each rainfall station. Instead of deriving separate frequency curves for each rainfall station to estimate weighted averages, a simpler, equivalent procedure was adopted. Each ranked weighted average point rainfall value was determined using the same modified Thiessen weighting procedure, followed by fitting an exponential distribution to provide estimates of the average point rainfalls for return periods from two to 20 years. The ARFs were then estimated directly using the corresponding areal and average point rainfall values associated with each return period or AEP (Bell 1976).

Bell's method, or modified versions thereof, have been used in several large-scale ARF studies conducted internationally, especially

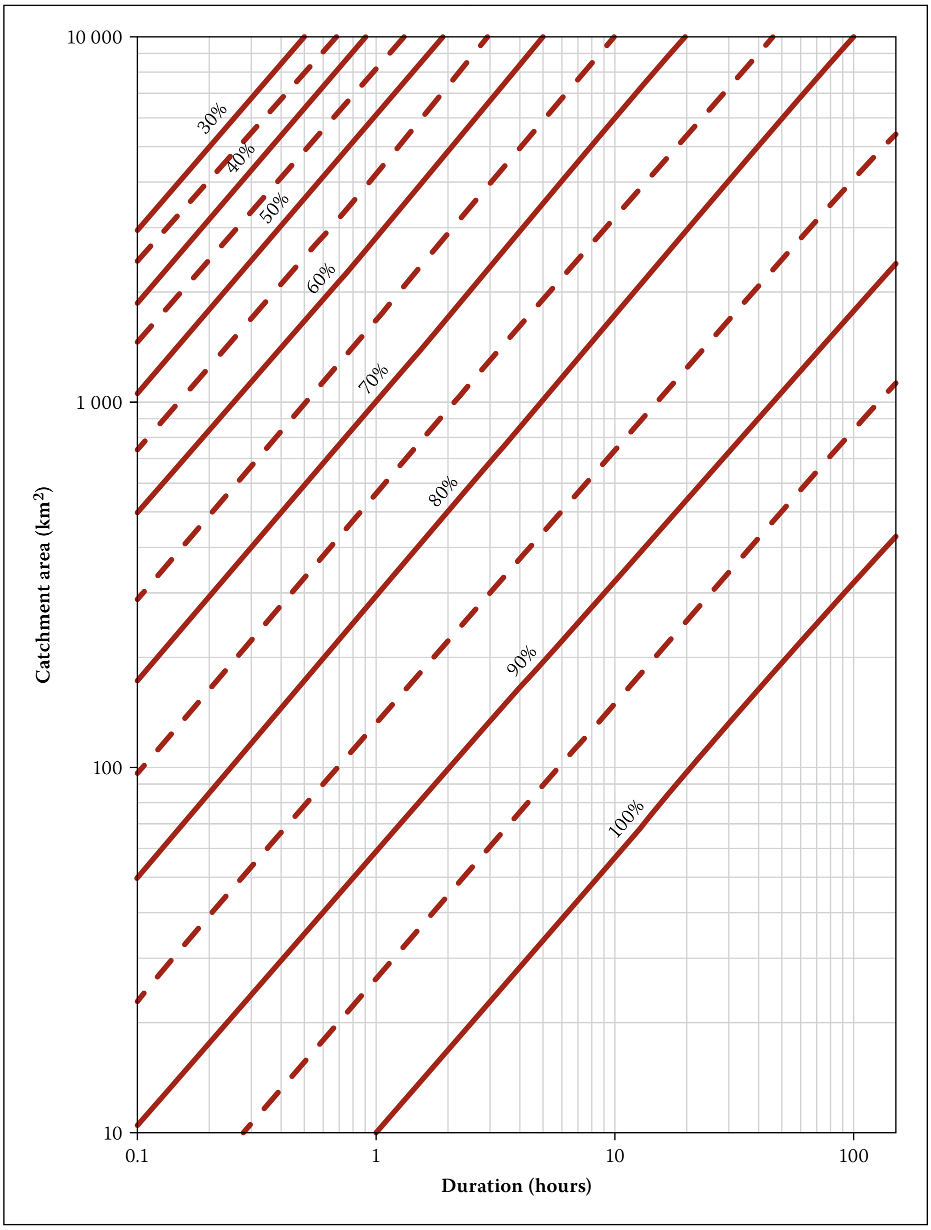

Figure 5 Revised ARF diagram for South Africa (Alexander 1990; 2001)

in the UK and Australia (Stewart 1989; Avery 1991; Masters 1993; Meynink & Brady 1993; Nittim 1989; Porter & Ladson 1993; Masters & Irish 1994; Siriwardena & Weinmann 1996). All these studies were conclusive that the basic procedure proposed by Bell (1976) is probabilistically more correct than other commonly used methods (e.g. USWB and UK FSR), since AEPs are explicitly included in the derivations of the ARFs.

Other international ARF estimation methods

Tables A1 and A2 in Appendix A provide the reader with a detailed summary of additional empirical and analytical ARF estimation methods used internationally.

STUDY AREA

The C5 secondary drainage region was selected as the pilot study area, since most of the required data is available from previous studies (Gericke & Du Plessis 2011; 2012) conducted in this region. South Africa is delineated into 22 primary drainage regions, which are further delineated into 148 secondary drainage regions. The pilot study area (Figure 6) is situated in primary drainage Region C and comprises the C5 secondary drainage region (Midgley *et al* 1994). The rainfall intensity in this region is normally high to very high with associated thunder activity and can be classified as convective rainfall. The average Mean Annual Precipitation (MAP) is 424 mm, ranging from 275 mm in the west to 685 mm in the east (Lynch 2004), and rainfall is characterised as highly variable and unpredictable. The rainy season starts in early September and ends in mid-April with a dry winter (Midgley *et al* 1994). Twelve gauged catchments, ranging from 38 km^2 to 33 277 km^2, were selected in the pilot study area to investigate the study objectives.

METHODOLOGY

This section provides the detailed methodology followed during this study which focuses on the evaluation and comparison of the numerical and graphical empirical ARF estimation methods currently used in South Africa in two distinctive phases.

First phase: Comparison of ARF estimation methods using standard input variables

The standard input variables and their associated ranges, e.g. catchment area (10 to 30 000 km^2), critical storm duration (one to 72 hours) and rainfall intensity (50 to 200 mm.h^{-1}) as depicted in Figures 1 and 2 were used as input with the mathematical algorithms derived from these ARF diagrams in order to assess the consistency between the numerical and graphical ARF estimation methods compiled by Van Wyk (1965) and Wiederhold (1969). In each case Microsoft Excel ARF diagrams were reproduced by manually extracting values from the original ARF diagrams (Figures 1–5). Thereafter the graphical results, as obtained from each reproduced ARF diagram, were compared to the ARF computed using the individual mathematical algorithms to highlight any biases and inconsistencies present. The Van Wyk (1965) and Wiederhold (1969) results were not included in the comparisons as these two methods are applicable to different areal ranges.

Second phase: Comparison of ARF estimation methods in the pilot study area

All the ARF estimation methods evaluated in Phase 1 were compared and evaluated in the 12 gauged catchments located in the C5 secondary drainage region described above to establish the biasness/consistencies/inconsistencies determined during Phase 1, as well as to establish the need for further research in this field. All the required catchment geomorphological variables (e.g. catchment area), time parameters (e.g. time of concentration/critical storm duration) and climatological variables (e.g. design rainfall depths and intensities) were obtained from Gericke & Du Plessis (2011). In this study, the design rainfall depths with return periods ranging from 10 to 200 years were based on the Regional Linear Moment Algorithm and Scale Invariance (RLMA&SI) approach developed by Smithers & Schulze (2003; 2004).

RESULTS AND DISCUSSION

The results based on the methodology used in this study are discussed below.

Review of ARF estimation methods

The estimation of ARFs based on either empirical or analytical estimation methods was evident from the literature review conducted. The review also highlighted that most of these methods could not be regarded as entirely true descriptions of the actual rainfall process, especially when the empirical methods, based on a limited amount of observed rainfall data, are applied outside their original developmental regions.

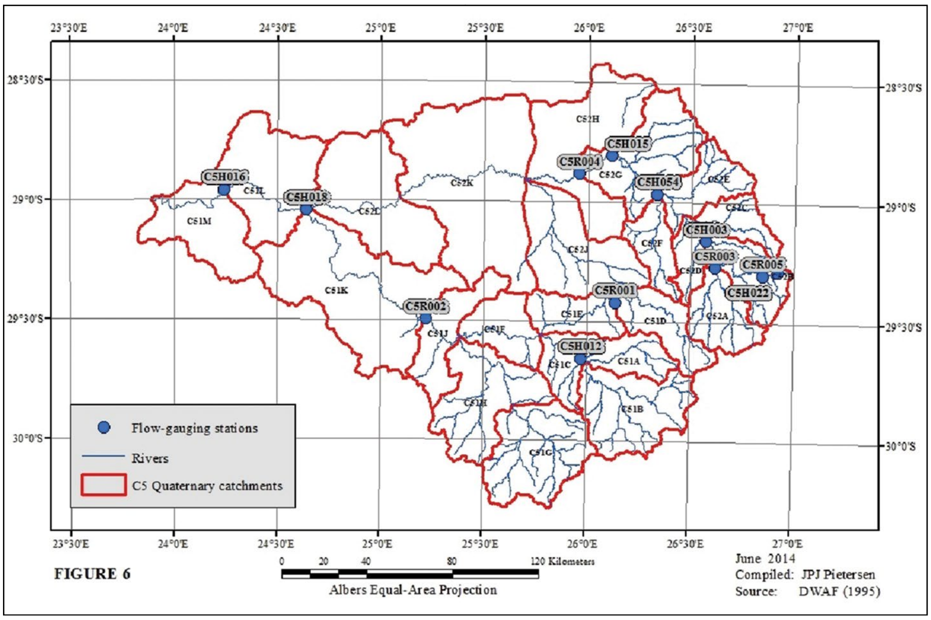

Figure 6 Location of study area in relation to the primary drainage regions of South Africa (Gericke & Du Plessis 2012)

Comparison of ARF estimation methods using standard input variables

As expected, all the ARF estimates decreased with an increase in catchment area, while significant differences, e.g. variation in the results obtained, also highlighted the presence of inconsistencies between the results from the numerical and graphical ARF estimation methods.

The comparison between Van Wyk's graphical (Figure 1) and numerical (Equation 1) results, as shown in Figure 7, is characterised by increasing averaged percentage differences associated with an increase in the catchment area, e.g. 7.1% (10 km^2), 7,8% (20 km^2), 12.2% (50 km^2), 18.3% (100 km^2), 23.8% (200 km^2), 27.3% (400 km^2) and 28% (800 km^2). A similar trend was also evident for the catchment area and rainfall intensities, in other words an increase in rainfall intensity associated with a specific catchment area resulted in larger percentage differences. However, despite these percentage differences, an overall coefficient of determination (r^2) of 0.96 confirmed the high degree of association between the ARFs estimated using the two methods. It is essential to keep in mind that different practising engineers might yield different ARF values when using graphical procedures to estimate ARFs, whereas the same values should be estimated using the numerical estimation method.

The comparison between ARF estimates using Wiederhold's graphical approach (Figure 2) and Equation 3, as shown in Figure 8, is characterised by a high degree of association (r^2 = 0.92) and increasing percentage differences associated with an increase in the catchment area, e.g. averaged differences of 1.1% (500 km^2), 1.7% (1 000 km^2), 8.0% (5 000 km^2), 13.6% (10 000 km^2), 22.5% (20 000 km^2) and 28.9% (30 000 km^2). In considering different storm durations associated with a specific catchment area, the

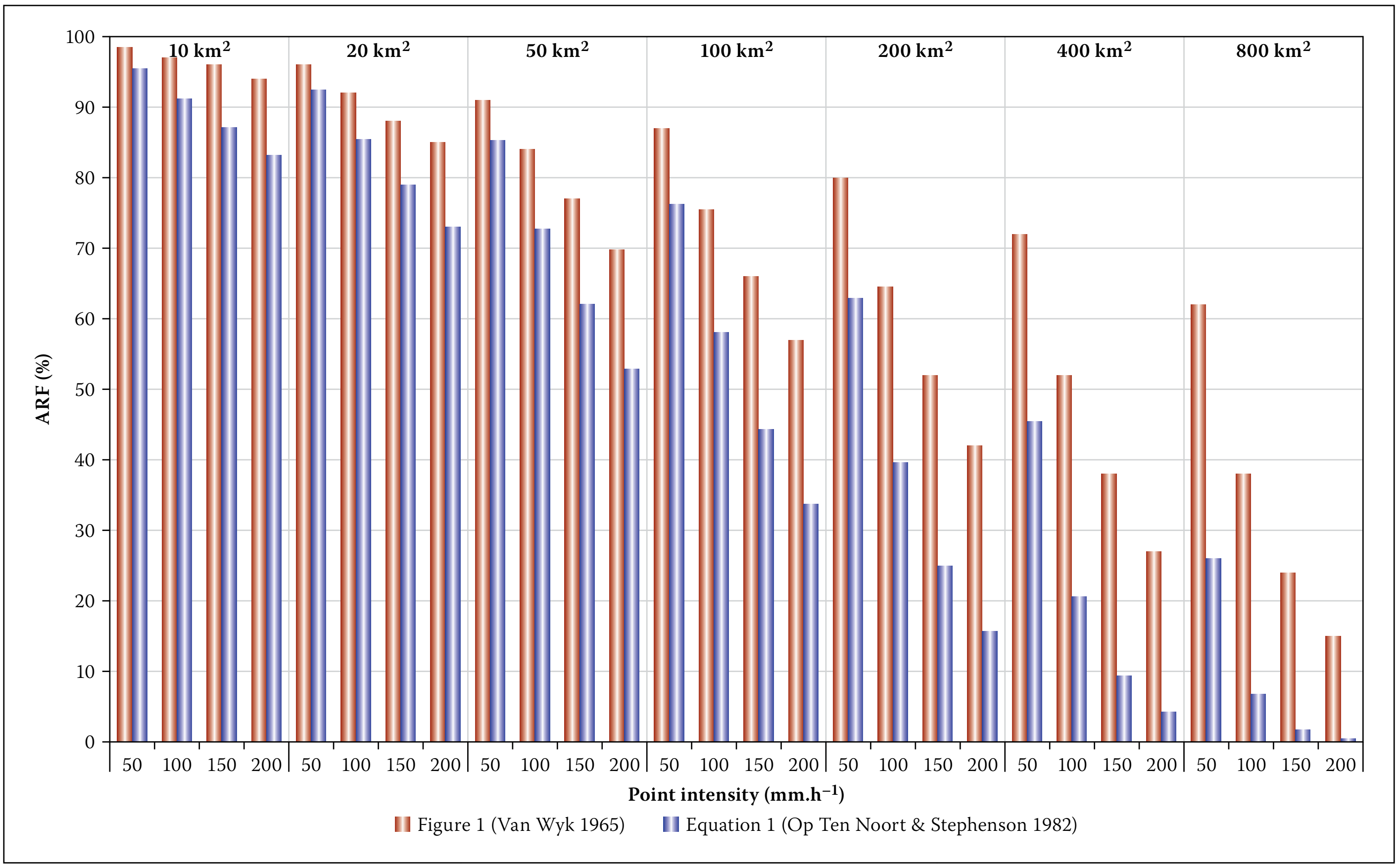

Figure 7 Comparison of the numerical vs graphical results (10 km^2 to 800 km^2) based on Van Wyk's (1965) storm-centred approach

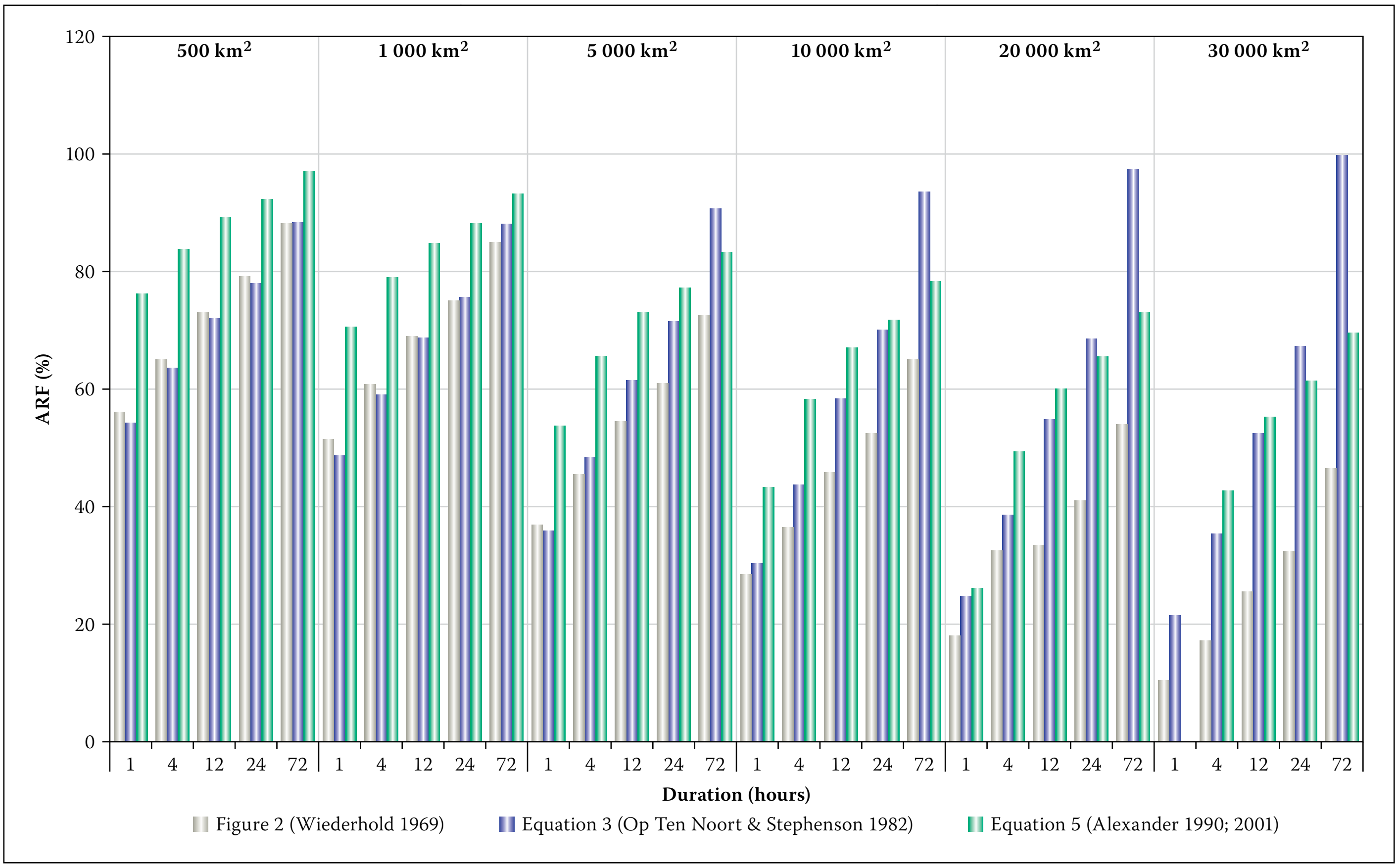

Figure 8 Comparison of the numerical vs graphical results (500 km^2 to 30 000 km^2) based on Wiederhold's (1969) and Alexander's (1990; 2001) approaches

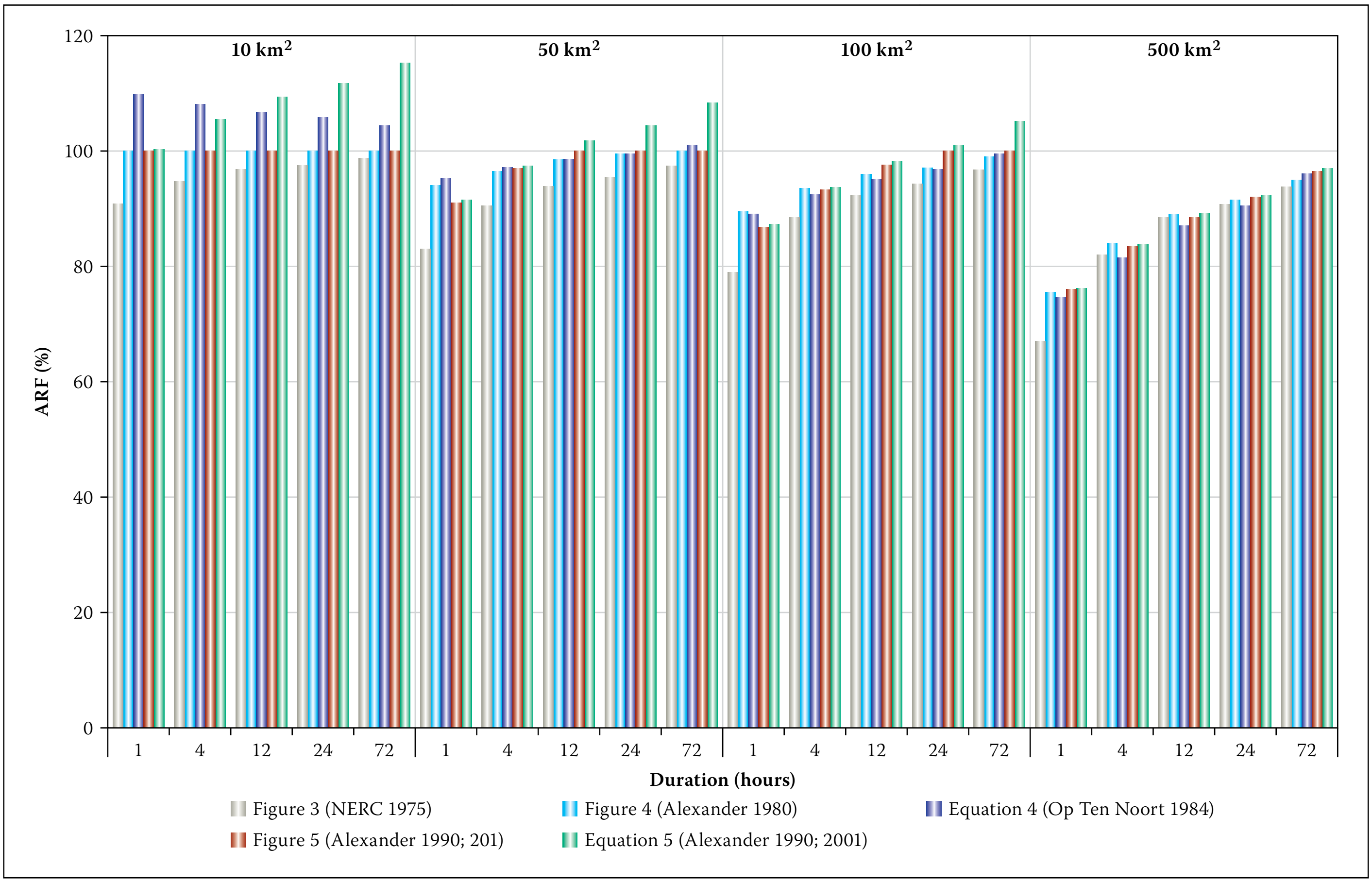

Figure 9 Comparison of the numerical vs graphical results (10 km^2 to 500 km^2) based on Alexander's (1980; 1990; 2001) geographically-centred approaches

ARF estimates increased with increasing storm duration.

Based on the above results, practising engineers should use the original graphical methods (Figures 1 and 2) as contained in the Drainage Manual (SANRAL 2013). However, it is important to note that both Van Wyk's and Wiederhold's methods are storm-centred empirical methods which are not suitable for estimating catchment areal design rainfall from design point rainfalls. In doing so, the practising

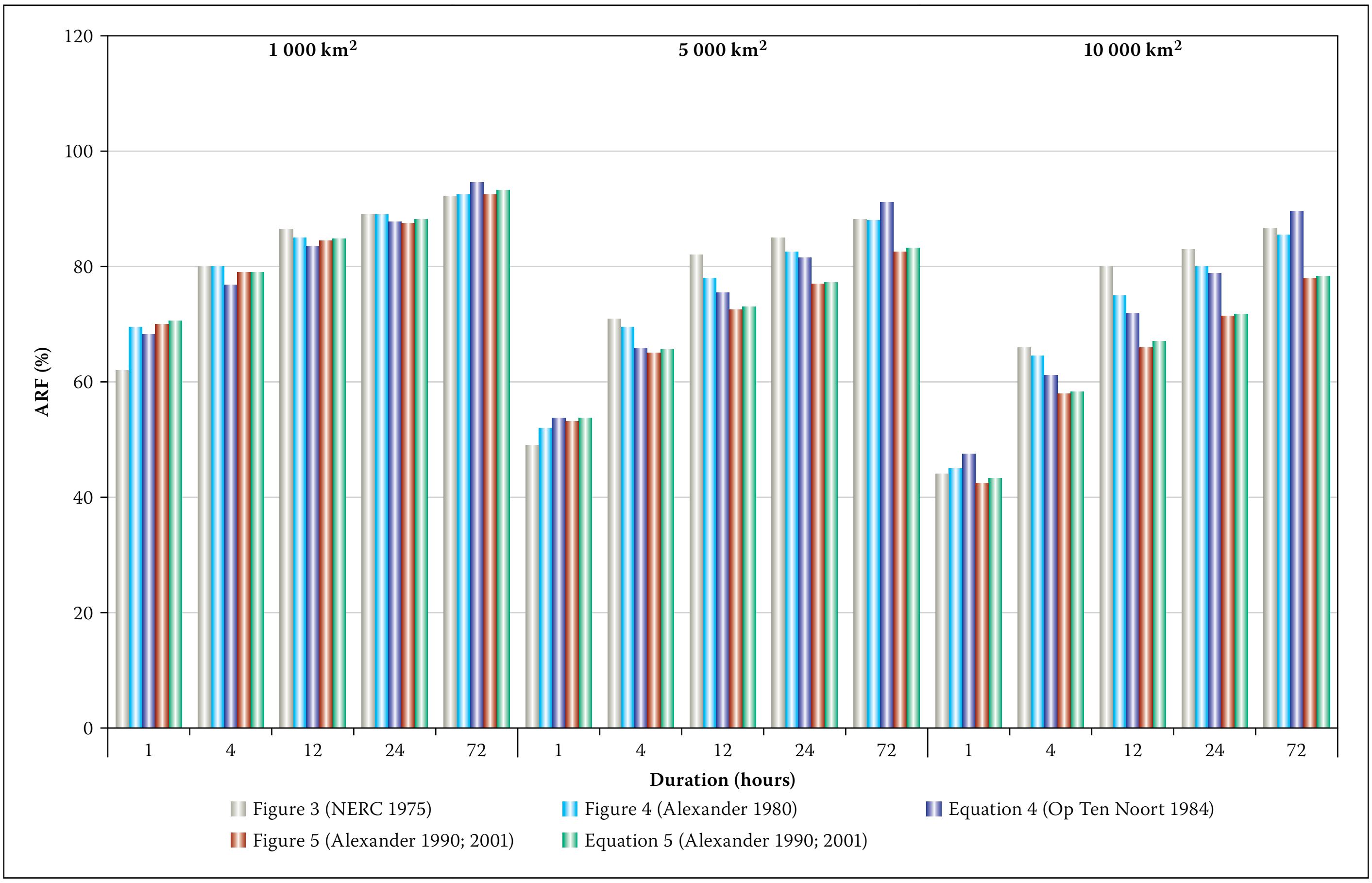

Figure 10 Comparison of the numerical vs graphical results (1 000 km^2 to 10 000 km^2) based on Alexander's (1980; 1990; 2001) geographically-centred approaches

engineer would by default incorrectly assume that extreme design point rainfall and extreme areal design rainfall are produced by the same rainfall event or rainfall type.

In the literature review it was highlighted that Alexander (1980) based his original methodology (Figure 4) on the UK FSR ARF diagrams (Figure 3, NERC 1975), from which Op ten Noort and Stephenson (1984) developed Equation 4. In applying these approaches, and Alexander's revised methodology (Figure 5 and Equation 5), the results shown in Figures 9 and 10 were obtained.

Typical average percentage differences varied from 2.2% to 5.6% (Figure 3 vs Figure 4); 0.2% to 6.9% (Figure 4 vs Equation 4) and 0.4 to 8.5% (Figure 5 vs Equation 5); 15.4% to 21.4% (Figure 2 vs Equation 5), with an overall tendency for larger percentage differences in the smaller catchment areas (10 km^2 ≤ A ≤ 100 km^2). The latter tendency also confirmed the findings of Alexander (1980), especially with specific reference when severe storm mechanisms produce very high intensity rainfall with cell core areas exceeding the areal range and storm duration under consideration. The coefficient of determination (r^2) results showed a high degree of association; 0.91 (Figure 3 vs Figure 4); 0.94 (Figure 4 vs Equation 4) and 0.96 (Figure 5 vs Equation 5); 0.87 (Figure 2 vs Equation 5). Based on these results it is evident that the geographically-centred numerical methods are generally more consistent, and this could likely also be one of the reasons why these methods are preferred to the storm-centred approaches, especially if multi-centred storms are to be considered.

Comparison of ARF estimation methods in the pilot study area

The application of the ARF estimation methods listed in Table 1 in the pilot study area showed some significant biases and systematic inconsistencies, which are summarised in Table 2.

The results contained in Table 2 are characterised by percentage differences ranging from 17.6% to 27.6% in the smaller catchments (38 km^2 to 937 km^2), 27.1% to 38% in medium sized catchments (1 650 km^2 to 6 331 km^2) and 44% to 71.5% in the large catchments (10 260 km^2 to 33 277 km^2). Similar to the results shown in Figures 9 and 10, these comparisons showed that the geographically-centred numerical ARF estimation methods are more consistent. However, the geographically-centred ARF estimates did not account for the variation of ARFs with return period.

CONCLUSIONS AND RECOMMENDATIONS

The comparison of the performance of ARF estimation methods over a range of input variables (e.g. catchment area, critical storm duration and rainfall intensity), and/or as applied in medium to large catchment areas in the C5 secondary drainage region in South Africa, highlighted that:

- The geographically-centred methods used in South Africa were either transposed from the UK with little local verification, or were developed using very limited local data. Hence, all these methods could potentially show significant variation from the observed areal rainfall characterising South African catchments, and thus ARFs developed from local rainfall data need to be developed.
- Ultimately, the significance of variation using different ARF estimation methods will only be appreciated when translated to design peak discharges. However, such an exercise is not possible at this stage of the study, but forms part of this high-priority on-going research area.

Based on the above it is evident that the ARFs currently used in South Africa need to be updated utilising the longer periods of records (40 years of additional data since the 1970s) which are now available for analysis. The variation of ARFs with return period

Table 2 ARF estimation results in the pilot study area (C5 secondary drainage region)

Catchment (area, km^2)	*Tc* (hours)	Return period (years)	Design rainfall depth (mm)	Rainfall intensity (mm.h^{-1})	ARF (%)									
					Figure 1	Equation 1	Figure 2	Equation 3	Figure 3	Figure 4	Equation 4	Figure 5	Equation 5	Equation 7
C5H022 (38)	1.6	10	50	31.2	95.5	92.3		76.9	87.5	96.0	98.2	95.0	95.1	96.1
		20	58	36.1	95.0	91.1								
		50	69	43.0	94.0	89.5								
		100	78	48.5	93.0	88.2								
		200	87	54.3	92.0	86.9								
C5R005 (116)	3.5	10	72	20.6	94.0	88.2		72.5	87.0	92.5	91.0	92.0	92.2	92.7
		20	84	23.9	92.0	86.4								
		50	100	28.7	91.0	84.0								
		100	114	32.4	90.0	82.0								
		200	128	36.4	89.0	80.1								
C5H054 (688)	16.9	10	78	4.6	96.0	89.6	74.0	73.6	88.8	89.0	87.3	88.5	88.8	86.9
		20	89	5.3	96.0	88.1								
		50	105	6.2	95.5	86.1								
		100	118	7.0	95.5	84.6								
		200	131	7.7	95.0	83.1								
C5R001 (922)	21.3	10	85	4.0		88.8	74.5	74.7	89.0	89.0	87.4	87.5	88.1	85.9
		20	98	4.6		87.1								
		50	117	5.5		84.8								
		100	132	6.2		83.1								
		200	147	6.9		81.2								
C5R003 (937)	13.9	10	81	5.8		83.8	70.5	70.4	87.4	86.5	84.8	86.0	86.0	85.8
		20	94	6.7		81.5								
		50	111	8.0		78.4								
		100	126	9.0		76.0								
		200	138	10.0		73.9								
C5H003 (1 650)	18.3	10	80	4.4		81.4	69.5	71.1	87.0	88.5	84.0	83.5	83.6	83.8
		20	93	5.1		78.8								
		50	109	6.0		75.6								
		100	122	6.7		73.1								
		200	135	7.4		70.7								
C5H012 (2 366)	20.2	10	77	3.8		79.0	67.0	71.0	86.5	84.5	83.2	81.5	81.7	82.5
		20	89	4.4		76.1								
		50	106	5.3		72.2								
		100	120	5.9		69.3								
		200	134	6.6		66.3								
C5H015 (6 009)	43.0	10	93	2.2		75.9	64.5	81.3	87.0	85.0	86.1	79.0	79.2	78.9
		20	108	2.5		72.8								
		50	127	3.0		68.7								
		100	142	3.3		65.8								
		200	157	3.7		62.9								
C5R004 (6 331)	47.9	10	90	1.9		77.9	66.0	83.4	87.0	85.0	86.9	79.5	79.4	78.7
		20	104	2.2		75.0								
		50	122	2.6		71.3								
		100	137	2.9		68.5								
		200	152	3.2		65.8								
C5R002 (10 260)	50.5	10	85	1.7		72.5	60.5	85.3			86.1		76.1	76.7
		20	98	1.9		68.9								
		50	117	2.3		64.2								
		100	132	2.6		60.6								
		200	147	2.9		57.1								
C5H018 (17 360)	99.6	10	115	1.2		71.7		106.7			91.9		76,1	74.5
		20	133	1.3		68.1								
		50	156	1.6		63.7								
		100	174	1.8		60.4								
		200	193	1.9		57.3								
C5H016 (33 277)	111.1	10	107	1.0		63.3		117.8			92.2		71.5	71.6
		20	123	1.1		59.0								
		50	145	1.3		53.7								
		100	162	1.5		49.9								
		200	180	1.6		46.3								

and with rainfall-producing mechanisms also needs to be investigated. It is envisaged that Bell's method (1976) based on a geographically-centred approach would be the most appropriate to use, since:

- the literature review conducted confirmed its large-scale preferential application internationally;
- the use of a geographically-centred approach would be most appropriate for a national-scale investigation bounded within a 'fixed' catchment area, e.g. at a quaternary catchment level; and
- currently in South Africa the storm-centered ARF methods (Van Wyk 1965 and Wiederhold 1969) are incorrectly applied in a geographically-centred manner, while assuming that extreme design point rainfall and extreme areal design rainfall are produced by the same rainfall event or rainfall type; given the use of point design rainfall to estimate ARFs, it is thus necessary to derive updated ARFs using a geographically-centred approach.

However, the use of a modified version of Bell's (1976) method, based on the annual maximum series (AMS) of point and areal rainfall as opposed to the PDS, is proposed. This modification would enable the development of probabilistically correct ARFs, namely the variation of ARFs with return period will be reflected, instead of using equally ranked observations curtailed to a common base period.

The final results should be presented in a suitable format to be useful to a practitioner, such as a set of ARF diagrams and associated algorithms.

REFERENCES

Alexander, W J R 1980. *Depth-area-duration-frequency properties of storm rainfall in South Africa.* Technical Report TR103. Pretoria: Department of Water Affairs.

Alexander, W J R 1990. Flood hydrology for southern Africa. Pretoria: SANCOLD.

Alexander, W J R 2001. *Flood risk reduction measures: Incorporating flood hydrology for southern Africa.* Pretoria: Department of Civil and Biosystems Engineering, University of Pretoria.

Allen, R J & DeGaetano, A T 2005. Areal reduction factors for two eastern United States regions with high rain-gauge density. *Journal of Hydrologic Engineering,* 10(4): 327–335.

Asquith, W H & Famiglietti, J S 2000. Rainfall areal-reduction factor estimation using the annual-maxima centered approach. *Journal of Hydrology,* 230(1–2): 55–69.

Avery, D R 1991. Design flood analysis in coastal New South Wales using areal average rainfall data. Paper presented at the Hydrology and Water Resources Symposium 1991, Perth, Australia, *IEAust. Nat. Conf. Pub. No.* 91/22: 627–628.

Bacchi, B & Ranzi, R 1996. On the derivation of the areal reduction factor of storms. *Atmospheric Research,* 42: 123–135.

Bell, F C 1976. *The areal reduction factor in rainfall frequency estimation.* Institute of Hydrology, Report No 35. Swindon, UK: Natural Environment Research Council.

Bengtsson, L & Niemczynowicz, J 1986. Areal reduction factors from rain movement. *Nordic Hydrology,* 17(2): 65–82.

De Michéle, C, Kottegoda, N T & Rosso, R 2001. The derivation of areal reduction factor of storm rainfall from its scaling properties. *Water Resources Research,* 37(12): 3247–3252.

Desbordes, M, Raous, P & Trévisiol, Y 1984. Areal reduction factors on short time and space intervals. *Water Science Technology,* (16): 189–198.

Gericke, O J & Du Plessis, J A 2011. Evaluation of critical storm duration rainfall estimates used in hydrology in South Africa. *Water SA,* 37(4): 453–470.

Gericke, OJ & Du Plessis, JA. 2012. Catchment parameter analysis in flood hydrology using GIS applications. *Journal of the South African Institution of Civil Engineering,* 54(2): 15–26.

Lambourne, J J & Stephenson, D 1986. *Research in urban hydrology and drainage: Factors affecting storm runoff in South Africa.* Water Systems Research Programme Report No 2/86, Johannesburg: University of the Witwatersrand.

Lombardo, F, Napoletano, F & Russo, F 2006. On the use of radar reflectivity for estimation of the areal reduction factor. *Natural Hazards and Earth System Sciences,* 6: 377–386.

Lynch, S D 2004. *Development of a raster database of annual, monthly and daily rainfall for southern Africa.* WRC Report No 1156/1/04, Pretoria: Water Research Commission.

Masters, C J 1993. *Areal reduction of point rainfall using Australian data.* MEng Sc Project Report. Sydney, Australia: School of Civil Engineering, University of New South Wales.

Masters, C J & Irish, J L 1994. Areal reduction factors for the Sydney region derived from spatial characteristics of heavy rainfall. Paper presented at the Hydrology and Water Resources Symposium 1994, Adelaide, Australia, *IEAust. Nat. Conf. Pub. No.* 94/15: 27–32.

Meynink W J C & Brady, D K 1993. Areal reduction factors for design storms. Paper presented at the Hydrology and Water Resources Symposium 1993, Newcastle, Australia, *IEAust. Nat. Conf. Pub. No.* 93/14: 257–271.

Midgley, D C, Pitman, W V & Middleton, B J 1994. *Surface water resources of South Africa.* Vol 2, Drainage Region C, Vaal: Appendices. Report No 298/2.1/9. Pretoria: Water Research Commission.

Myers, V A & Zehr, R M 1980. *A methodology for point-to-area rainfall frequency ratios.* NOAA Technical Report No 24. Washington DC: United States Department of Commerce, National Oceanic and Atmospheric Administration, National Weather Services.

NERC (Natural Environment Research Council) 1975. *Flood studies report.* Swindon, UK: NERC.

Nittim, R 1989. Areal distribution of design rainfall. Paper presented at the Hydrology and Water Resources Symposium 1989, Christchurch, New Zealand, *IEAust. Nat. Conf. Pub. No.* 89/19: 487–488.

Omolayo, A S 1989. *Relationships between point and areal rainfall for flood and drought assessments.* Unpublished PhD thesis, Sydney, Australia: University of New South Wales.

Omolayo, A S 1993. On the transposition of areal reduction factors for rainfall frequency estimation. *Journal of Hydrology,* 145(1–2): 191–205.

Op ten Noort, T H & Stephenson, D 1982. *Flood peak calculation in South Africa.* Water Systems Research Programme Report No 2/1982. Johannesburg: University of the Witwatersrand.

Porter, J W & Ladson A R 1993. An application of fixed-area rainfall reduction factors. Paper presented at the Hydrology and Water Resources Symposium 1993, Newcastle, Australia, *IEAust. Nat. Conf. Pub. No.* 93/14: 273–279.

Rodriguez-Iturbe, I & Mejia, J M 1974. On the transformation of point rainfall to areal rainfall. *Water Resources Research,* 10(4): 729–735.

SANRAL (South African National Roads Agency Limited) 2013. *Drainage manual*, 6th ed. Pretoria: SANRAL.

Siriwardena, L & Weinmann, P E 1996. *Derivation of areal reduction factors for design rainfalls in Victoria.* Report No 96/4. Victoria, Australia: Cooperative Research Centre for Catchment Hydrology.

Sivapalan, M & Blöschl, G 1998. Transformation of point rainfall to areal rainfall: Intensity-duration-frequency curves. *Journal of Hydrology,* 204(1–4): 150–167.

Skaugen, T 1997. Classification of rainfall into small- and large-scale events by statistical pattern recognition. *Journal of Hydrology,* 200(1–4): 40–57.

Smithers, J C & Schulze, R E 2003. *Design rainfall and flood estimation in South Africa.* WRC Report 1060/01/03. Pretoria: Water Research Commission.

Smithers, J C & Schulze, R E 2004. The estimation of design rainfall for South Africa using a regional scale invariant approach. *Water SA,* 30(4): 435–444.

Stewart, E J 1989. Areal reduction factors for design storm construction: Joint use of rain gauge and radar data. *Association of Hydrological Sciences,* 181: 31–40.

Svensson, C & Jones, D A 2010. Review of methods for deriving areal reduction factors. *Journal of Flood Risk Management,* 3(3): 232–245.

USWB (United States Weather Bureau) 1957. *Rainfall intensity-frequency regime, the Ohio Valley.* Technical Paper 29. Washington DC: United States Department of Commerce.

USWB (United States Weather Bureau) 1958. *Rainfall intensity-frequency regime, south-eastern United States.* Technical Paper 29. Washington DC: United States Department of Commerce.

Van Wyk, W 1965. *Aids to the prediction of extreme floods from small watersheds*. Johannesburg: Department of Civil Engineering, University of the Witwatersrand.

Wiederhold, J F A 1969. *Design storm determination in South Africa*. HRU Report No 1/1969. Johannesburg: Hydrological Research Unit, University of the Witwatersrand.

Yoo, C, Kim, K, Kim, H S & Park, M J 2007. Estimation of areal reduction factors using a mixed gamma distribution. *Journal of Hydrology*, 335(3–4): 271–284.

APPENDIX

Table A1 Summary of empirical ARF estimation methods used internationally

Approach	Method	Mathematical algorithm	Origin	Comments
Geographically-centred	**USWB method** (USWB 1957; 1958)	$ARF = \frac{N\sum_{i=1}^{N}\sum_{j=1}^{n} w_i \overline{\overline{P}}_{ij}}{\sum_{i=1}^{N}\sum_{j=1}^{n} P_{ij}}$ where: ARF = areal reduction factor N = number of stations within the catchment area n = record length (years) $\overline{\overline{P}}_{ij}$ = point rainfall for station i on the day of the annual maximum areal rainfall in year j (mm) P_{ij} = annual maximum point rainfall of station i in year j (mm) w_i = Thiessen weighted factor for station i	USA	■ Observed rainfall records (10 to 15 years of data) from dense rainfall monitoring networks in catchment areas (250 km^2 to 1 000 km^2) were used. ■ Rainfall record lengths were regarded as insufficient to establish the effect of return period/AEP on the point-area rainfall relationships. ■ The areal rainfall of each event and associated duration was estimated using Thiessen weights. ■ The mean of the AMS was estimated, while the highest point rainfall measurement at each station in a particular year was selected.
Geographically-centred	**UK FSR method** (NERC 1975)	$ARF = \frac{1}{nN}\sum_{i=1}^{N}\sum_{j=1}^{n}\left(\frac{\overline{\overline{P}}_{ij}}{P_{ij}}\right)$ where: ARF = areal reduction factor N = number of stations within the catchment area n = record length (years) $\overline{\overline{P}}_{ij}$ = point rainfall for station i on the day of the annual maximum areal rainfall in year j (mm) P_{ij} = annual maximum point rainfall of station i in year j (mm)	UK	■ Thirteen catchment areas (10 km^2 to 18 000 km^2) and storm durations ranging from 2 minutes to 25 days were used. ■ Nation-wide UK rainfall records were used for the development of an ARF estimation diagram with catchment area and storm duration as variables. ■ ARF values were assumed to fit an average recurrence interval of between 2 to 3 years; however, return period/AEP was not taken into account, since the effect thereof was regarded as insignificant.
Geographically-centred	**Bell's method** (Bell 1976)	$ARF_m = \frac{\sum_{i=1}^{N}(w_i \overline{\overline{P}}_{ij})_m}{\sum_{i=1}^{N}(w_i P_{ij})_m}$ where: ARF_m = areal reduction factor (ratio of areal rainfall of rank m to the Thiessen weighted average point rainfall of the same rank (%) m = rank value N = number of stations within the catchment area $\overline{\overline{P}}_{ij}$ = point rainfall for station i on the day of the annual maximum areal rainfall in year j (mm) P_{ij} = annual maximum point rainfall of station i in year j (mm) w_i = ratio of the areal rainfall of rank m to the Thiessen weighted average point rainfall of the same rank	UK	■ Based on the derivation of frequency curves of areal and average point rainfall. ■ Estimate ARFs from the ratio of areal to average point rainfall at the relevant AEPs. ■ Areal rainfall is determined from Thiessen weights of the annual maximum point rainfall values. ■ More probabilistically correct ARFs compared to the USWB, NERC and the Desbordes *et al* (1984) methods. ■ Dependant on the return period. ■ Significantly lower ARFs for high AEPs (20–100 years) were obtained. ■ This method showed a tendency towards lower ARFs with longer AEPs for shorter duration (24 hour and less) rainfall events.
Geographically-centred	**Stewart's method** (Stewart 1989)	$ARF_T = \frac{P_{AS}(T)\overline{P_A}}{P_{PS}(T)\overline{P_P}}$ where: ARF_T = areal reduction factor at a specific AEP (%) $\overline{P_A}$ = mean of annual maximum areal rainfall (mm) $P_{AS(T)}$ = standardised T-year areal rainfall (mm) $\overline{P_P}$ = mean of annual maximum point rainfall (mm) $P_{PS(T)}$ = standardised T-year point rainfall (mm)	UK	■ Based on Bell's method (1976) using daily rainfall data from north-west England. ■ A total of 834 rainfall stations with at least 25 years of data were used. ■ A total of 544 sample catchments (25 km^2 to 10 000 km^2) and storm durations ranging from 1 day to 8 days were analysed. ■ ARFs were expressed as a function of the geographical location and AEP. ■ ARFs decreased with an increasing catchment area and AEP. ■ ARF estimates proved to be significantly lower than those based on the UK FSR method (NERC 1975).

Approach	Method	Mathematical algorithm	Origin	Comments
Geographically-centred	**Omolayo's method** (Omolayo 1993)	$ARF = \frac{P_A(A,T_d,T)}{\frac{1}{\sum_i w_i}\sum_i [w_i \overline{P_P}\ (Td,T)]}$ where: ARF = areal reduction factor A = catchment area under consideration (km²) n = record length (years) P_A = T-year areal rainfall (mm) $\overline{P_P}$ = average T-year point rainfall (mm) T_d = storm duration (hours) w_i = weighted average P_P of the gauges i in the same region	Australia	■ Daily rainfall data (30 years record length) was used. ■ The 1-day ARFs for the USA were transposed to Australia, given that the climatological variables were similar. ■ Probabilistically correct ARF estimation. ■ ARF is defined as the ratio between areal rainfall and point rainfall of the same return period/AEP. ■ Point rainfall used is assumed to be representative for the entire catchment. ■ Data intensive method.
Geographically-centred	**Modified Bell's method** (Siriwardena & Weinmann 1996)	$ARF_m = 1 - 0.4(\mathrm{A}^{0.14} - 0.7\log T_d)T_d^{0.48} + 0.002A^{0.4}T_d^{0.41}\left[0.3 + \log\left(\frac{1}{T}\right)\right]$ where: ARF_m = areal reduction factor A = catchment area (km²) T = return period (years) T_d = storm duration (hours) **Ranges of application:** 1 km² ≤ A ≤ 10 000 km² 0.05 ≤ AEP ≤ 0.0005 18 hours ≤ T_d ≤ 120 hours	Australia	■ AMS of areal and point rainfall were used instead of the PDS curtailed to a common base period as originally proposed by Bell (1976). ■ Over 2 000 daily rainfall stations in Victoria, Australia, were used. ■ ARF values were estimated for a number of 'circular sample catchment areas' distributed through areas characterised by a high-density rainfall-monitoring network. ■ ARF values were estimated for rainfall durations (1 to 3 days), catchment areas (125, 250, 500, 1 000, 4 000 and 8 000 km²) and return periods (2 to 200 years).
Geographically-centred	**Mixed gamma distribution method** (Yoo *et al* 2007)	$ARF_{(A,T)} = 1 - Me^{-(aA^b)^{-1}}$ where: ARF = areal reduction factor A = rainfall storm areas (km²) M, a, b = parameters associated with each return period T = return period (years)	Korea	■ A total catchment area of 9 843 km² containing 25 rainfall stations with at least 30 years of records were used. ■ Method utilises daily rainfall data instead of probabilistic curve fitting of the AMS. ■ Return periods ranging from 2 years to 1 000 years were considered. ■ ARF estimates are based on 1-day storm durations.
Storm-centred	**Annual maxima-centred method** (Asquith & Famiglietti 2000)	$ARF = \frac{\int_0^R 2rS_{T(r)}\Delta r}{R^2}$ where: ARF = areal reduction factor A = rainfall storm areas (km²) R = maximum radius of circular catchment or integration limit (km) r = radius of concentric circle within the catchment (km) $S_{T(r)}$ = ratio between rainfall depth at a specific location, distance r from the point of the design storm and the annual maxima rainfall	USA	■ Method developed for the Austin, Dallas and Houston regions, USA, with a dense rainfall-monitoring network. ■ The Austin region (15 600 km²) had 108 daily rainfall stations, Dallas region (21 000 km²) had 103 daily rainfall stations and Houston region (35 800 km²) had 193 daily rainfall stations. ■ Several record lengths exceeded 80 years. ■ Method focuses on the analysis of the areal rainfall distribution to estimate ARFs for design storms. ■ ARFs decrease rapidly with increasing AEPs.
Analytical-empirical	**National Weather Service method** (Myers & Zehr 1980; cited by Svensson & Jones 2010)	$ARF = \frac{\overline{P_A}(f,\Delta t,A)}{\overline{P_P}(f,\Delta t,0)}$ where: ARF = areal reduction factor $\overline{P_A}$ = average areal rainfall for a specific frequency (f), duration (Δt) and area (A) (mm) $\overline{P_P}$ = point rainfall for a specific frequency (f), duration (Δt) and area (A) (mm)	USA	■ Method is based on the probabilistic analysis of rainfall AMS pair values of individual stations and the distance between these stations. ■ Rainfall depth-area curves were developed from a dense rainfall-monitoring network. ■ Effect of return period/AEP on ARFs is included, i.e. probabilistically correct ARFs. ■ ARFs decrease with increasing return periods. ■ ARFs not regarded as representative of the spatial and temporal rainfall variability. ■ Very complex approach and difficult to implement in practice.

Table A2 Summary of analytical ARF estimation methods used internationally

Approach	Method	Mathematical algorithm	Origin	Comments
Spatial correlation	**Rodriquez-Iturbe-Mejia method** (Rodriquez-Iturbe & Mejia 1974; cited by Svensson & Jones 2010)	$ARF = \sqrt{E(\rho(d))}$ where: ARF = areal reduction factor $E\ (\rho(d))$ = expected correlation coefficient for the characteristic correlation distance.	Various	- Simple ARF estimation approach used in various areas. - Based on a spatial correlation structure using either an exponentially decaying function or a Bessel-type correlation structure. - Dependent on all observed rainfall data, i.e. the primary data and not only the AMS. - 'Design storm' areal rainfall distributions are not included.
Storm movement	**Storm movement method** (Bengtsson & Niemczynowicz 1986)	$ARF = \frac{L_P}{L} = \frac{vT_d}{L} \qquad \text{if } L_P < 0.5$ $ARF = 1 - \frac{0.25L}{L_P} = 1 - \left(\frac{0.25L}{L_P}\right) \qquad \text{if } L_P \geq 0.5$ where: ARF = areal reduction factor L = catchment length (km) L_P = extension of block rain cell (km) T_d = storm duration (hours) v = storm speed (m.s^{-1})	Sweden	- Represents the relationship between rainfall movement and ARFs. - ARFs are based on the limited extension of rain cells, movement and spacing between rain cells and the effect of rain cells on one another. - ARFs were obtained from point rainfall hyetographs and storm speeds. - Relations were established between moving storm-derived ARFs and ARFs estimated by a dense rainfall-monitoring network. - ARFs proved to be constant in Norway.
Spatial correlation	**Omolayo's method** (Omolayo 1989; cited by Svensson & Jones 2010)	LN distributed rainfall: $ARF_1 = Exp\left\{K_T\sigma\left(\sqrt{\frac{1+(N-1)\rho}{N}} - 1\right)\right\}$ Normal distributed rainfall: $ARF_2 = \sqrt{\frac{1+(N-1)\rho}{N}}$ Normal distributed rainfall (large number of rainfall stations): $ARF_3 = \sqrt{\rho}$ where: ARF = areal reduction factor K_T = frequency factor corresponding to return period N = number of rainfall stations T = return period (years) σ = standard deviation of rainfall depth in the log domain (mm) ρ = average spatial correlation coefficient	Australia and USA	- Based on the average spatial correlation and the number of rainfall stations within an area. - Rainfall depths are assumed to be log-normally distributed. - Return period is considered. - The normal distribution expression is similar to the relationship derived by Rodriguez-Iturbe & Mejia (1974), except that the correlation coefficient is averaged over the rainfall stations. - ARFs vary directly with the spatial correlation coefficient and inversely with standard deviation, number of rainfall stations and AEPs.
Crossing properties	**Bacchi-Ranzi method** (Bacchi & Ranzi 1996)	$ARF_{(A,Td,F)} = \frac{T_{A,Td}(F')}{T_A(F')}$ where: ARF = areal reduction factor A = area under consideration (km^2) F' = F-quantile of the corresponding probability distribution T_d = duration within the space-time domain where the rainfall process can be assumed uniform (hours) T = return period (years)	Italy	- Sixteen Constant Altitude Plan Position Indicator (CAPPI) maps were recorded and analysed from the C-band weather radar to be compared with the corresponding rainfall data from 17 rainfall stations. - Based on the analysis of the crossing properties of the spatial and temporal rainfall process. - High rainfall intensity processes were assumed to be Poisson distributed. - ARF expressed as the ratio of areal and point rainfall intensity values associated with the same duration and frequency. - ARFs are dependent on the return period and catchment area.
Spatial correlation	**Sivapalan-Blöschl method** (Sivapalan & Blöschl 1998)	$ARF\left[k^2\left(\frac{A}{\lambda^2}\right),T_d,T\right] = \frac{b(T_d)c(T_d)k^2F_2(k^{-2}) - \frac{T_{A,Td}(F')}{T_A(F')}\ln\left[\ln\left(\frac{T}{T-1}\right)\right]}{b(T_d)c(T_d) - \ln\left[\ln\left(\frac{T}{T-1}\right)\right]}$ where: ARF = areal reduction factor A = catchment area (km^2) b = function of duration, where $b(T_d) = -0.05 + 0.25T_d^{0.49}$ c = function of duration, where $c(T_d) = 0.2 + 20T_d^{-0.7}$ $F_1(k^{-2})$ = generic properties of the gamma distribution $F_2(k^{-2})$ = generic properties of the gamma distribution k^2 = rainfall correlation structure T = return period (years) T_d = storm duration (hours) λ = spatial correlation length (km)	Austria	- Based on a spatial correlation structure using both extreme value and/or parent distributions. - ARF values are dependent on the catchment area, storm duration (spatial correlation structure) and return period. - The ARF values are independent of the rainfall regime. - ARF values decrease with an increasing catchment area and return period. - Method is rather regarded as a 'geographically-centred' method as opposed to 'storm-centred'. - The final ARF expression is regarded as complex and not user-friendly.

Approach	Method	Mathematical algorithm	Origin	Comments
Scaling relationship	**De Michéle's method** (De Michéle *et al* 2001; cited by Svensson & Jones 2010)	$ARF = \left[1 + \omega\left(\frac{A^z}{T_d}\right)^b\right]^{\frac{\nu}{b}}$ where: ARF = areal reduction factor A = catchment area (excluding the rain gauge area) (km²) T_d = storm duration (hours) b, v, ω, z = fitted parameters	Italy	■ Only eight years of rainfall data were used. ■ Storm durations (20 minutes to 6 hours) and catchment areas (0.25 km² to 300 km²) were used. ■ Return periods or AEPs were not included/considered. ■ Method proved to be most reliable for storm durations between 1 hour and 3 hours, while less satisfactory for 20 minute and 6 hour storm durations. ■ Kriging was used to estimate the rainfall intensity AMS.
Radar data	**Polar 55C method** (Lombardo *et al* 2006)	$ARF_{(Td,T)} = \frac{i_A(T_d,T)}{i_{A=1}(T_d,T)}$ where: ARF = areal reduction factor A = area under consideration (km²) i = rainfall intensity (mm.h^{-1}) T = return period (years) T_d = storm duration (hours)	Italy	■ The ARF values were estimated by using radar reflectivity maps collected with Polar 55C. ■ Rainfall intensities over the radar scanning region (allowing a single radar image to last for one minute) were estimated for durations (1, 5, 10, 60 and 120 minutes) and return periods (2, 10, 25 and 50 years) by using the Arithmetic mean and Thiessen polygon methods. ■ The radar rainfall estimates were integrated for heavy rainfall data over an area of 900 km². ■ The radar used in this study is located 15 km south-east of Rome. ■ Study focused on the influences of area, storm duration, intensity and return period on ARF variation. ■ The ARFs exceeded unity in small areas characterised by relatively longer storm durations.